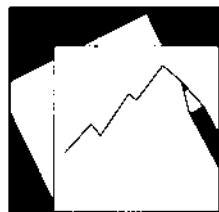


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Globalization, the Business Cycle, and Macroeconomic Monitoring

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IMF Working Paper

Research Department

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Abstract

We propose and implement a framework for characterizing and monitoring the global business cycle. Our framework utilizes high-frequency data, allows us to account for a potentially large amount of missing observations, and is designed to facilitate the updating of global activity estimates as data are released and revisions become available. We apply the framework to the G-7 countries and study various aspects of national and global business cycles, obtaining three main results. First, our measure of the global business cycle, the common G-7 real activity factor, explains a significant amount of cross-country variation and tracks the major global cyclical events of the past forty years. Second, the common G-7 factor and the idiosyncratic country factors play different roles at different times in shaping national economic activity. Finally, the degree of G-7 business cycle synchronization among country factors has changed over time.

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1 Introduction

In the modern environment of radically enhanced global macroeconomic and financial linkages, isolated country analysis seems highly insufficient for informed assessment of the state of real activity, and hence for informed decision making. Hence we propose and implement a framework for characterizing and monitoring the *global* business cycle. Our framework is informed by economic theory and structured so as to help inform subsequent economic theory. We apply it to the G-7 countries, and in so doing we extend the empirical research program on the global business cycle along several dimensions.

First, we consider the roles played by a large set of macroeconomic indicators when we construct our country and global cycles. The country and global factors that we estimate provide a better characterization of business cycles as they encompass a wide array of activity measures, in the tradition of Burns and Mitchell (1946) and much subsequent research. This contrasts with most of the literature on global business cycles, which uses only quarterly national income and product account data.

Second, our comparatively comprehensive set of indicators enables us to provide a systematic characterization of global and national business cycles. In particular, we analyze various statistical properties of cycles, and we relate certain cyclical episodes to the movements in country and global macroeconomic factors. We also study the interaction of activity across countries and with the global cycle.

Third, and related, we use our rich set of indicators to explore the *evolution* of the global business cycle. We emphasize, among other things, whether and how cross-country business cycle synchronization has evolved in response to the forces of globalization. Against this background, we devote special attention to the recent recession.

We proceed as follows. In Section 2 we review several literatures that bear on our concerns. We first provide a summary of various empirical approaches used to model the global business cycle. Then, considering that our measures of global and national business cycles should help us analyze the evolution of business cycle synchronization, we also review the literature on linkages between globalization and synchronization. The main message is that, although various approaches have been employed, it has been a challenge to construct practical and satisfactory tools for monitoring global business cycles.

In Section 3 we construct and examine a new G-7 dataset, which contains a variety of real activity indicators. In particular, we use six widely-followed real activity indicators for each country whenever available: employment, GDP, disposable income, industrial production, retail sales, and initial claims for unemployment insurance. Because the indicators

are available at different frequencies and dates, they provide valuable and complementary high-frequency information about the state of the economy.

In Section 4 we introduce and fit a simple dynamic factor model for real activity separately for each country. We work in a state space framework with multiple indicators and a single latent activity factor, which we extract optimally using the Kalman filter. One distinguishing feature of our approach is that we are able to utilize mixed-frequency data, specifying the model at high frequency and allowing for a potentially large amount of missing data (for the less-frequently observed variables). The country factors that we extract explain most of the common variation in underlying country activity indicators, and they are consistent with a number of well-known business-cycle episodes in each country. Moreover, we find that the degree of country-factor synchronization has changed over time in response to growing global linkages, which change the importance of common vs. country-specific shocks.

In Section 5 we estimate a hierarchical multi-country model. After obtaining the estimated country factors, we decompose their movements into those coming from a common G-7 factor and those coming from idiosyncratic components. The G-7 factor measures the global business cycle, capturing common fluctuations in country factors, which are themselves reflections of common movements in underlying activity variables in each country. The G-7 factor captures a significant amount of common variation across countries and reflects the major cyclical events of the past forty years. Moreover, it appears to play different roles at different times in shaping national economic activity. We conclude in Section 6.

2 An Interpretive Literature Review

Here we present a brief and selective survey of empirical strategies used to model the global business cycle as relevant for our subsequent development, examining how those strategies have evolved in academic and policy circles over the years. We pay special attention to the links, both theoretical and empirical, between globalization and business-cycle synchronization.

2.1 Empirical Modeling of the Global Business Cycle

As global linkages have become stronger, the interest in understanding the dynamics of global activity has increased in both academic and policy circles. Many studies use simple measures of global activity, which are often based on a country size weighted average of the major advanced countries' output growth (see Ahmed et al. (1993)). More recently, developments

of new econometric methods and advances in computing technology have facilitated the use of more sophisticated approaches, such as dynamic factor models.¹ These models have been quite successful in capturing common fluctuations in multiple time-series of a large cross-section of countries. Some of these models rely on a single measure of aggregate activity, such as output, while others employ multiple indicators, including output, consumption and investment, in order to provide more reliable estimates of global business cycles (see Gregory et al. (1997); Kose et al. (2003a); Kose et al. (2003b); and Kose et al. (2008b)).² As we present later in this section, these models have been widely used to study the evolution of global business cycles.

In policy circles as well, there has been an increasing appreciation of the importance of well-designed tools to track global economic activity. Approaches employed by policy institutions differ considerably in technical sophistication and scope. The International Monetary Fund (IMF), for example, uses a simple country size weighted average of each member country's output growth rate to arrive at its estimate of the world output growth.³ Given that the IMF membership includes a rather diverse set of 187 countries, the measure it employs provides a simple and intuitive characterization of global economic activity. However, the measure has also some drawbacks. First, GDP is often available only at quarterly frequency making it difficult to monitor global activity at higher frequencies. Second, as much as it is a simple and intuitive measure, it is based on a single indicator, GDP, which is a rather crude measure of activity with a variety of well-known shortcomings.

In addition to the simple measures mentioned above, applications of various composite, leading, and coincident indicators have been employed to assess the state of activity in a (functional/regional) group of countries. Well-known examples of these include the OECD's composite leading indicators and the CEPR's EuroCOIN. Both indicators are available in monthly frequency and employ a large number of activity variables. The OECD's composite leading indicators use various weighting and filtering methods to aggregate information from the underlying activity variables.⁴ The indicators are intended to provide early signals of

¹One of the earliest contributions to this literature is Stock and Watson (1989) who employ a dynamic factor model to develop a composite index of coincident indicators for the United States.

²Some other studies focus on models to derive the likelihood of the phases of global business cycle using dynamic factor models combined with Markov switching methods (see Chauvet and Yu (2006)).

³To be more specific, the IMF uses weights that correspond to GDP valued at purchasing power parity (PPP) as a share of total world GDP (see IMF (April 2010)).

⁴The OECD methodology closely follows the techniques developed by the NBER. Burns and Mitchell (1946) and Moore and Shiskin (1967) put together the very early versions of the composite indexes of leading, coincident, and lagging indicators of economic activity. The Conference Board uses the same framework to produce its various indexes of U.S. economic activity.

turning points in business cycles of various groups of countries, including the OECD area, euro area, Major Five Asia, and G-7. However, like most other composite indexes of activity, the OECD's indexes also lack a well-defined econometric methodology and involve a rather subjective determination of the underlying economic variables and their aggregation.

The CEPR's EuroCOIN is a coincident indicator designed to monitor euro area activity in real time. The index is estimated using a Generalized Dynamic Factor Model (see Altissimo et al. (2001)). Using a sizable number of data series, including measures of real and financial sectors activity and surveys of business and consumer sentiment, the indicator provides an estimate of the monthly growth of Euro area GDP.

As the discussion so far has shown, although various approaches have been employed, it has been a challenge to construct practical and satisfactory tools to monitor global business cycles. The methodology we use in this paper has several advantages over existing approaches. First, our framework is useful for monitoring global economic activity in real-time. Second, our measure of global business cycles captures common movements in a wide range of indicators, such as GDP, income, retail sales, initial claims, employment, and industrial production. We combine the information content of activity measures available at different frequencies (monthly as well as quarterly) to arrive at a monthly measure. Third, our measure of global activity is obtained using linear and exact procedures that are easily reproducible. Fourth, our methodology leads to a coherent analysis of interactions between the global business cycle and country-specific cycles as it employs a well-defined hierarchical structure to estimate these cycles.

2.2 Globalization and Business Cycle Synchronization

A large literature examines the implications of globalization, which is often associated with increased international trade and financial linkages, for the synchronization of international business cycles.

2.2.1 Theory

Economic theory has ambiguous predictions about the impact of increased trade and financial linkages on the comovement amongst macroeconomic aggregates across countries. Stronger trade linkages can lead to higher or lower degree of comovement depending on the nature of integration and the form of specialization patterns. International trade linkages generate both demand and supply-side spillovers across countries, which can increase the

degree of business cycle synchronization. For example, on the demand side, an investment or consumption boom in one country can generate increased demand for imports, boosting economies abroad. On the supply-side, a positive tradable output shock leads to lower prices; hence, imported inputs for other countries become cheaper. Through these types of spillover effects, stronger international trade linkages can result in more highly correlated business cycles across countries.

However, both classical and “new” trade theories imply that increased openness (trade linkages) to trade leads to increased specialization. How does increased specialization affect the degree of synchronization? The answer depends on the nature of specialization (intra- vs. inter-industry) and the types of shocks (common vs. country-specific). If stronger trade linkages are associated with increased inter-industry specialization across countries, then the impact of increased trade depends on the nature of shocks: If industry-specific shocks are more important in driving business cycles, then international business cycle comovement is expected to decrease. If common shocks, which might be associated with changes in demand and/or supply conditions, are more dominant than industry specific shocks, then this would lead to a higher degree of business cycle comovement.

What about the impact of financial integration on the extent of business cycle comovement? Analytically, the effects of financial integration also depend on the nature of shocks and the form of specialization patterns. For example, financial linkages could result in a higher degree of business cycle synchronization by generating large demand side effects as the changes in equity prices affect the dynamics of wealth. If consumers from different countries have a significant fraction of their investments in a particular stock market, then a decline in that stock market could induce a simultaneous decline in the demand for consumption and investment goods in these countries because of its impact on domestic wealth. Furthermore, contagion effects that are transmitted through financial linkages could also result in heightened cross-country spillovers of macroeconomic fluctuations.

However, international financial linkages could decrease the cross-country output correlations as they stimulate specialization of production through the reallocation of capital in a manner consistent with countries’ comparative advantage in the production of different goods. For example, Kalemli-Ozcan et al. (2003) find that there is a significant positive correlation between the degree of financial integration (risk sharing) and specialization of production. In other words, through increasing financial linkages countries can have a more diversified portfolio and are able to insure themselves against idiosyncratic shocks. This would lead to less correlated cross-country fluctuations in output as it could result in more

exposure to industry- or country-specific shocks. However, since such specialization of production would typically be expected to be accompanied by the use of international financial markets to diversify consumption risk, it should result in stronger comovement of consumption across countries.

Increased integration could also affect the dynamics of comovement by changing the nature and frequency of shocks. First, as trade and financial linkages get stronger, the need for a higher degree of policy coordination might increase, which, in turn, raise the correlations between shocks associated with nation specific fiscal and/or monetary policies. This would naturally have a positive impact on the degree of business cycle synchronization. However, it is not clear, at least in theory, whether increasing trade and financial linkages indeed lead to a growing need for the implementation of coordinated policies. Traditional arguments, based on trade multiplier models, would suggest that increased linkages implies a growing need for international policy coordination (see Oudiz and Sachs (1984)). However, recent research by Obstfeld and Rogoff (2003) provides results quite different than those in the previous literature. They argue that integration may in fact diminish the need for policy coordination since international capital markets generate an expanded set of opportunities for cross-country risk sharing.

Second, shocks pertaining to changes in productivity could become more correlated, if increased trade and financial integration leads to an acceleration in knowledge and productivity spillovers across countries (see Coe and Helpman (1995)). More financially integrated economies are able to attract relatively large foreign direct investment flows which have the potential to generate productivity spillovers.

Third, increased financial integration and developments in communication technologies lead to faster dissemination of news shocks in financial markets. This could have a positive impact on the degree of business cycle synchronization if, for example, good news about the future of domestic economy would increase domestic consumption through its impact on wealth, and, if consumers in other countries, who hold stocks in the domestic country, raise demand for goods in their countries. In other words, shocks associated with news, which are rapidly transmitted in global financial markets, could lead to a higher degree of interdependence across economic activity in different countries.

2.2.2 Empirics

Empirical studies are also unable to provide a concrete explanation for the impact of stronger trade and financial linkages on the nature of business cycles. There has been a growing re-

search program examining the empirical relationship between increased linkages and the dynamics of business cycle comovement using a variety of methods. A widely popular approach in this literature involves with the study of the changes in some simple measures of business cycle comovement over time. Another strand of the literature directly examines how increasing trade and financial linkages affect the business cycle correlations employing various regression models. A third approach uses recently developed econometric methods, such as dynamic factor models, to examine the characteristics of common factors in business cycles.

The studies in the first group focus on the evolution of comovement properties of the main macroeconomic aggregates over time in response to changes in the volume of trade and financial flows. The results of these studies indicate that differences in country coverage, sample periods, aggregation methods used to create country groups, and econometric methods employed could lead to diverse conclusions about the temporal evolution of business cycle synchronization. For example, some of these studies find evidence of declining output correlations among industrial economies over the last three decades. Helbling and Bayoumi (2003) find that correlation coefficients between the United States and other G-7 countries for the period 1973-2001 are substantially lower than those for 1973-1989. In a related paper, Heathcote and Perri (2004) document that the correlations of output, consumption, and investment between the U.S. and an aggregate of Europe, Canada, and Japan are lower in the period 1986-2000 than in 1972-1985. Results by Doyle and Faust (2005) indicate that there is no significant change in the correlations between the growth rate of output in the United States and in other G-7 countries over time.

The empirical studies in the second group employ cross-country or cross-region panel regressions to assess the role of global linkages on the comovement properties of business cycles in developed and developing countries. While Imbs (2004) and Imbs (2006) find that the extent of financial linkages, sectoral similarity, and the volume of intra-industry trade all have a positive impact on business cycle correlations, Baxter and Kouparitsas (2005) and Otto et al. (2001) document that international trade is the most important transmission channel of business cycles. The results by Kose et al. (2003b) suggest that both trade and financial linkages have a positive impact on cross-country output and consumption correlations. Calderon et al. (2007) report that international trade linkages lead to higher cross-country business cycle correlations among developed countries than developing countries.

Some other studies employ factor models to study the changes in the degree of business cycle comovement, but those studies also report conflicting findings. Stock and Watson

(2005) employ a factor-structural VAR model to analyze the importance of international factors in explaining business cycles in the G-7 countries since 1960. They conclude that comovement has fallen in the 1984-2002 period relative to 1960-1983 due to diminished importance of common shocks. Kose et al. (2008b) employ a Bayesian dynamic factor model to analyze the evolution of comovement since 1960.⁵ Using the data of the G-7 countries, they document that the common (G-7) factor on average explains a larger fraction of output, consumption, and investment volatility in the globalization period (1986-2005) than that in the 1960-1972 period. They interpret this result as an indication of increasing degree of business cycle synchronization in the age of globalization. Kose et al. (2008a) employ a dynamic factor model to analyze the evolution of synchronization in a large sample of industrial, emerging and developing countries. They report that since the mid-1980s there has been a higher degree of synchronization of business cycle fluctuations among the group of industrial economies and among the group of emerging market economies.

3 A G-7 Real Activity Indicator Dataset

We work with a G-7 dataset. Although the G-7 is a smaller group of countries than we ultimately hope to incorporate, it is nevertheless highly-significant and certainly much more encompassing than the U.S. alone. Indeed the U.S. is responsible for only twenty-five percent of world real output at market exchange rates, whereas the G-7 is responsible for fifty percent.⁶

Partly reflecting our desire to maximize transparency and convenience, and partly reflecting the paucity of useful and comparable high-frequency real activity indicator data available for a large group of countries, we adopt a monthly base frequency. This eliminates many of the complications in Aruoba et al. (2009), including time-varying system matrices, high-dimensional state vectors, etc.

For each country we use data matching six economic concepts where available: employment, GDP, disposable income, industrial production, retail sales and initial claims for unemployment insurance. There are several reasons why we focus on those variables. First, they constitute an integral part of activity indexes that are used to study the direction of the economy by policy institutions, think tanks, and financial markets. While GDP is the

⁵There is a rich literature on large dynamic factor models (see, e.g., Forni et al. (2000); Forni and Reichlin (2001), Stock and Watson (2002); Doz et al. (2008)).

⁶These approximate shares are for 2006-2008; see IMF (April 2010). Eventually we hope to move to G-20 and beyond. The G-20 covers roughly ninety percent of global economic activity; see www.G20.org.

most widely followed indicator of aggregate activity, the others move closely with the different phases of the cycle. Initial claims is a leading indicator of business cycle; industrial production, retail sales, and income are coincident indicators; and employment is a lagging indicator. Second, the business cycle dating committees, including the NBER Business Cycle Dating Committee and CEPR Business Cycle Dating Committee, base their decisions on those or closely-related indicators. Third, and related, those variables are the ones used to produce the ADS Business Conditions Index based on Aruoba et al. (2009) now provided by the Federal Reserve Bank of Philadelphia, as well as the Conference Board's composite coincident index, among others. We use the same set of variables here, to the extent possible, for the other countries.

We gathered all data in April 2010; the resulting sample ranges from 1970 through 2009. Across the seven countries, we have a total of 37 series observed over (at most) 40 years. The specific statistical series available sometimes differ across countries, although the economic concepts measured are highly similar. For example, we use U.S. payroll employment and Canadian civilian employment. Sources vary, but we rely heavily on the Haver and OECD databases. We measure all indicators in real terms. We use indicators seasonally adjusted by the relevant reporting agency.⁷ We transform all indicators to logarithmic changes, except initial claims; hence all are flows.

We summarize certain aspects of the data in Table 1, which gives for each country the series used, the data source, and the data range. We also indicate implicitly in the range the observational frequency of each indicator; some are monthly ("M") and some are quarterly ("Q"). The frequency differs not only across series, but sometimes also for the same series across countries. For example, French employment is measured quarterly, whereas German employment is measured monthly. Finally, we remove extreme outliers (more than four standard deviations from the mean) from all series.⁸

In Figures 1-7 we plot the indicators for each country expressed in annual growth terms, except for initial claims which is reported as a fraction of the labor force for most countries. They are quite noisy – month-to-month indicator movements in industrial production and retail sales, for example, can occasionally be very large when expressed at annual rates. It is clear that idiosyncratic noise in the individual indicators masks much of the real activity information contained in them, which is of course the reason for using several indicators for each country. In what follows we use optimal filtering methods that effectively calculate

⁷There are two exceptions: Canadian and Japanese initial unemployment claims are reported in seasonally unadjusted form, so we seasonally adjust the data ourselves using the Census X-12 algorithm.

⁸This results in deleting a total of 29 observations across all countries and periods.

sophisticated averages across idiosyncratic indicators, eliminating much of the noise and producing accurate assessments of underlying real activity.

4 Single-Country Real Activity Modeling

For each country we observe a variety of indicators, all of which contain information about the latent state of economic activity. Hence, building on earlier work of Stock and Watson (1989), Mariano and Murasawa (2003), Aruoba et al. (2009) and Aruoba and Diebold (2010), we work in a state space framework with multiple indicators and a single latent activity factor, which we extract optimally using the Kalman filter. We allow for mixed-frequency data, specifying the model at high frequency (in the present case, monthly) and allowing for a potentially large amount of missing data (for variables that are less-frequently-observed and/or variables more recently available). In what follows we first discuss the country factor model with some precision, and then we present a variety of empirical results, both within and across countries.

4.1 A Single-Country Dynamic Factor Model

The latent real activity factor x_t evolves monthly with covariance-stationary autoregressive dynamics,

$$x_t = \rho_1 x_{t-1} + \dots + \rho_p x_{t-p} + \eta_t, \quad (1)$$

where η_t is a white noise innovation with unit variance. The i -th covariance-stationary daily indicator \hat{y}_t^i may depend linearly on x_t and a measurement error ε_t^i :

$$\hat{y}_t^i = c^i + \beta^i x_t + \varepsilon_t^i, \quad (2)$$

with

$$\varepsilon_t^i = \gamma_1^i \varepsilon_{t-1}^i + \gamma_2^i \varepsilon_{t-2}^i + \dots + \gamma_q^i \varepsilon_{t-q}^i + v_t^i. \quad (3)$$

v_t^i are white-noise shocks that are uncorrelated with each other and with η_t . In our implementation below, we use $p = q = 3$.

Note that some indicators, although evolving monthly, are not *observed* monthly. If y_t^i denotes \hat{y}_t^i observed at a possibly lower frequency, then the relationship between y_t^i and \hat{y}_t^i depends on whether \hat{y}_t^i is a stock or flow variable as well as the frequency of observation. Remember that all of our observed variables are flow variables either because they are orig-

inally flow variables as in initial claims or because we use their growth rates. For variables observed every month, we have $y_t^i = \hat{y}_t^i$ and (2) is the measurement equation. For variables observed every quarter, the measurement equation is

$$y_t^i = \begin{cases} \sum_{j=0}^2 \hat{y}_{t-j}^i = 3c^i + \beta^i (x_t + x_{t-1} + x_{t-2}) + (\varepsilon_t^i + \varepsilon_{t-1}^i + \varepsilon_{t-2}^i) & \text{if } y_t^i \text{ is observed} \\ NA & \text{otherwise.} \end{cases} \quad (4)$$

Compiling the components, our framework corresponds to a state space system:

$$\boldsymbol{\alpha}_{t+1} = \mathbf{T}\boldsymbol{\alpha}_t + \mathbf{R}\mathbf{u}_t \quad (5)$$

$$\mathbf{y}_t = \mathbf{c} + \mathbf{Z}\boldsymbol{\alpha}_t, \quad (6)$$

for $t = 1, \dots, \mathcal{T}$, where $\boldsymbol{\alpha}_t$ is a vector of state variables which contains appropriate lags of x_t and ε_t^i , \mathbf{y}_t is a vector of observed variables, \mathbf{u}_t collects the innovations η_t and v_t^i , \mathbf{c} collects the constant terms and \mathcal{T} denotes sample size. The innovations \mathbf{u}_t are distributed according to $\mathbf{u}_t \sim (0, \mathbf{Q})$. In Appendix A we show the exact state space object we use with the associated matrices.⁹

Importantly for us, despite the missing data the Kalman filter and associated likelihood evaluation via prediction-error decomposition remain valid in our environment, subject to some simple modifications. This is well-known, as discussed for example in Durbin and Koopman (2001) and exploited in Aruoba et al. (2009). The benefit of this approach is that we can use simple modifications of the standard Kalman filter and smoother to produce exact maximum-likelihood estimates of our model, and to produce optimal estimates of its latent macroeconomic activity factor, x_t . We now proceed to do so.

4.2 Single-Country Empirics and Extraction of Country Real Activity

In this section, we first present our estimation results with respect to the country factors and discuss how they relate to the underlying activity variables. Then, we briefly describe the temporal evolution of the country factors in order to evaluate whether they are consistent with the well-known historical episodes of business cycles. Next, we analyze the extent of

⁹To identify the factor model, we normalize the variance of η such that x_t has a unit variance. This requires a non-linear restriction in $\rho_1, \rho_2, \dots, \rho_p$ and the variance of η .

synchronization across factors and the evolution of synchronization over time.

We report three main results in this sub-section. First, the country factors we estimate capture most of the common variation in underlying activity indicators. Second, they are able to capture the main macroeconomic developments over the past 40 years as they clearly feature periods of recessions and expansions that are fairly consistent with the business cycle narrative of these countries. In particular, they show that the 2008-2009 recession is the longest and deepest episode in a number of countries. Third, although the extent of co-movement across countries is quite high over the full sample, it varies over time. While it is likely that increased globalization of trade and financial linkages explains some of the temporal movements in the degree of co-movement, changing intensity of common and country-specific shocks also appears to play an important role.

4.2.1 Country Factors

We estimate our country factors using the measurement equations (6) and transition equations (5). The estimated measurement equations, reported in Table 2, top panel, reveal that almost all indicators (34 out of 37) for all countries load positively and significantly on the country factors. The only exceptions are initial claims in the U.S. and Canada, as expected given its counter-cyclical nature – with negative loadings – and in Japan and France – with statistically insignificant coefficients. Disposable income in Italy is not statistically significant suggesting that this variable is unable to provide useful information to characterize the Italian business cycles.

The estimated transition equations, Table 2, middle and bottom panels, typically reveal significant positive serial correlation in the dynamics of country factors. The extracted factor is essentially a measure of deviation from the mean growth rate, so it is not necessarily expected to feature an extremely high serial correlation. The dominant roots of the autoregressive lag operator polynomials are smaller than one for all countries, with all but one country featuring complex roots.

We extract the country factors using the Kalman smoother. The extent of contemporaneous co-movement between country factors and the underlying indicators of activity varies across countries (see panel (a) of Table 3). For instance, the co-movement between the country factors and GDP is quite high, ranging from a low of 0.61 for Germany to a high of 0.89 for France. In contrast, the co-movement between the country factors and some other indicators, such as retail sales and income, tends to be relatively low. In Canada, the United States, and the United Kingdom, the country factors are also highly correlated

with both industrial production and employment. Perhaps somewhat surprisingly, the U.S. country factor features higher co-movement with these two activity variables than it does with GDP. In fact, among the six activity indicators, GDP features the highest correlation with the country factor in all countries except the U.S. Confirming the findings from the measurement equations, initial claims are negatively and weakly correlated with the country factors.

In order to further examine the links between the country factors and underlying activity variables, we regress each country factor obtained through the Kalman filter on current and 12 lags of standardized activity variables. By construction, the filtered factor is a linear function of the current and all lagged indicators. By running this regression, we essentially recover the weights of each indicator at each lag.¹⁰ Given the properties of the Kalman filter, the filtered factor is necessarily a linear function of all current and past observables, implying that one would theoretically expect an R^2 of unity if we had a large enough sample. For most of the countries, we indeed have very high R^2 numbers reflecting that these 13 lags are enough to capture most of the variation for the factors. Since both the left-hand-side and the right-hand-side variables are such that they have zero mean and unit standard deviation, the coefficients can be compared across activity variables. In panel (b) of Table 3 we report the coefficient of the contemporaneous activity variables and in panel (c) we report the sum of the coefficients for each variable to get its total effect. The results once again emphasize the tight connection between the country factors and GDP and industrial production. In particular, GDP appears to be the most important variable in driving country factors in all cases except the U.S. where industrial production appears to be the most influential. After GDP, industrial production tends to be the most relevant one while both income and initial claims have minor roles in explaining the country factors.

4.2.2 Evolution of Country Factors

How successful are our country factors in capturing the well-known episodes of business cycles since 1970? This is an important question since we later use these factors to estimate our global factor that is intended to be used to monitor global economic activity in real-time. In order to answer this question, we present the estimated country factors in the top panels of Figures 8-14. We discuss the bottom panels of these figures in Section 5.

¹⁰Stock and Watson (1988) do a similar exercise with their coincident indicator. In their case, since all indicators are monthly and there is no temporal aggregation/missing observation problems, they are able to obtain these weights directly from the estimated system matrices. In our case, the said problems complicate matters and these regressions provide a good approximation.

Note that in the estimation we normalized the variance of each country factor to unity and the magnitudes of the factors can be interpreted as the distance from the mean (of zero) in number of standard deviations: for example if the factor is equal to -2 , it is 2 standard deviations below the mean, and given the normality assumption, just outside the 95% confidence bands around zero. The country factors are generally quite successful in displaying the main macroeconomic developments in the G-7 countries.¹¹ In particular, the country factors clearly feature periods of recessions and expansions that are fairly consistent with the business cycle narrative of each of these countries. Well known recessionary episodes such as the downturns of the mid-1970s, early 1980s, early 1990s, early 2000s, and the latest wave of recessions in 2008-2009 are clearly captured by the country factors.

Moreover, the country factors are also able replicate the expansionary periods of the late 1970s, mid-1980s, the long-expansion during the 1990s, and the growth acceleration of the mid-2000s. In particular, several of the peaks and troughs of the U.S. country factor coincide with the NBER reference cycle dates.¹² In addition, the U.S. factor features the growth accelerations of the early 1970s, 1983-1985, and the mid-1990s. Somewhat surprisingly, the U.S. country factor suggests that the recessions of 1975 and 1980 are deeper than the 2008-2009 recession, a pattern not observed just by looking at output cycles. A closer inspection of the underlying data, however, indicates that the adverse effects on employment and industrial production of the 1975 recession are indeed much more pronounced than those of the 2008-2009 recession.¹³

The Japanese country factor displays the evolutionary dynamics of domestic business cycles. In particular, the factor for Japan shows two distinctive decades—the boom decades of the 1970s and 1980s, during which Japan switched to an information-based export economy, and the "Lost Decade" of the 1990s, that resulted from the collapse of asset price bubble in the late 1980s. The well-documented episodes of recessions are also clearly visible. For example, Japan is very much affected by the oil shock in the 1970s due to its heavy dependence on imported oil. The Japanese economy went through brief periods of recessions in the 1980s and early 2000s. The impact of the latest recession on Japan is particularly severe because

¹¹Claessens et al. (2009) and Claessens et al. (2010) examine the main features of recessions and recoveries in advanced countries.

¹²The NBER reference business cycle dates: Troughs: Feb. 1961, November 1970, March 1975, July 1980, November 1982, March 1991, and November 2001. Peaks: April 1960, December 1969, November 1973, January 1980, July 1981, July 1990, March 2001, and December 2007. For these dates, see the NBER web page.

¹³As Aruoba and Diebold (2010) also demonstrate, the significant aspect of the 2008-2009 recession for the U.S. was its combined severity – its length and depth – relative to the other recessions. Using this metric, it is by far the worst recession for the U.S. in our sample.

of the adverse effects of the synchronized collapse of global trade on export industries in Japan (see Sommer (2009)). For the sake of brevity, we do not discuss the details for each country, but it is obvious that the factors we estimate are consistent with the evolutions of national cycles.

The country factors also provide evidence that the 2008-09 recession is the longest and deepest in Germany, Italy, the U.K, and Japan. In the case of France, however, the 2008-09 recession is as deep as the 1974-75 recession, although the former one is twice as long. Lastly in Canada, the country factor suggests that the 1981-83 recession is the worst one over the past 40 years, however, the 2008-09 is the longest one. To emphasize the depth of the 2008-2009 recession, Table 4 reports the months with the lowest values for each country's factor. 35 out of 70 entries belong to the 2008-2009 period, which are denoted with boldface. The second period which is significant across all countries is the 1974-1975 recession with 20 entries, denoted in italics. In addition, the country factors are able to capture the dynamics of the ongoing recoveries. The factors suggest that the recovery started in early 2009, with the U.S. leading the rebound. By the end of 2009, Canada, Japan and the U.S. already registered positive levels of activity while the European countries still experienced relatively weaker recoveries.

The evolution of the volatility of country factors has declined over time, especially from the mid-1980s until the global financial crisis of the 2008-2009. In particular, the average volatility has declined by half from the 1970s to the period 2000-07. This suggests that our country factors clearly capture the Great Moderation phenomenon, characterized by the large decline in macroeconomic volatility, widely documented in the literature (see, for instance, Blanchard and Simon (2001) and Stock and Watson (2003)). Although there is still a debate about the underlying causes of the decline in volatility, the Great Recession has changed the nature of this debate as it probably marked an end to the era of the Great Moderation. Inclusion of the years 2008 and 2009 into the sample leads to higher volatility in the 2000s than those in the 1980s and 1990s (but still smaller than that in the 1970s), thus interrupting the downward trend in volatility associated with the Great Moderation. (not reported)

4.2.3 Synchronization of Country Factors

We next examine the extent of synchronization of business cycles using cross-country correlations of factors. The top left panel of Table 5 reports the correlations for the full sample. These correlations vary substantially across countries as they range between a low of 0.25

(Italy and the U.K.) and a high of 0.64 (France-Germany). The neighboring country-pairs, with stronger trade and financial linkages, exhibit relatively higher correlations: The country factors of Canada and the U.S. in North America and those of France, Germany, and Italy in Europe, for example, feature the highest correlations. In addition, the degree of synchronization between the country factors of Germany and Japan is quite high (0.59) probably because of the relatively high export dependence of these countries making them sensitive to global economic developments.

We also compare cross-country correlations of factors with those of underlying indicators, including output, income, and industrial production (not reported). This is a useful exercise considering that most of the literature on the synchronization of business cycles does not go beyond the usual aggregates, such as GDP and IP. Our larger set of activity indicators provides a broader perspective about the extent of business cycle synchronization. The exercise yields a number of interesting observations. First, cross-country correlations of indicators tend to be lower than those of factors. This is an intuitively appealing result since factors are representations of common fluctuations across the underlying indicators.

Second, there are some qualitative similarities as well as differences between correlations of factors and those of the underlying indicators. In the case of output, for example, the country-pair with the lowest correlation is the same as that of factor-pair. However, the country pair with the highest output correlation is the pair of France-Italy while the France-Germany pair, which has the highest in the case of factors, is not even in the top three. Similar observations extend to the comparisons of cross-country correlations of income and industrial production with those of factors. These findings suggest that the use of factors brings out information about the extent of synchronization of business cycles that is not available by the analysis of correlations of indicators.

How does the extent of synchronization of country factors change over time? As we summarize in section II, this question is at the heart of the literature analyzing the linkages between globalization and synchronization of national business cycles. Since our country factors provide aggregate measures of national business cycles, it is useful to examine the evolution of business cycle synchronization using them. In the remaining panels of Table 5, we report pairwise correlations for each decade in our sample. Our simple measure of synchronization is the average of cross-country correlations in each decade.

By this metric, there has been a gradual decline in the degree of synchronization from the 1970s to the 1990s. However, this observation does not necessarily indicate a negative link between globalization and synchronization since the extent of synchronization also depends

on the commonality of shocks. For example, the high degree of synchronization in the 1970s (relative to the 1990s), probably reflects the impact of oil price shocks of the era. In the 1980s, the average correlation is similar to that of the 1970s. The common shocks associated with the tight monetary policies adopted by a number of the G-7 countries in the early 1980s probably play a role on the cross-country correlations during that decade.

The decade with the lowest average correlation is the 1990s. However, this largely reflects Japan's "Lost Decade" and the slow growth in Germany after the unification. It is likely that these country-specific developments lead to relatively low correlations of these countries' factors with others. For example, when we exclude Japan, the average correlation in the 1990s rises to 0.32, and when we exclude both Japan and Germany, the average goes up to 0.43.

The degree of synchronization increases to its highest level, roughly 0.74, in the 2000s. However, this increase is mostly due to the 2008-09 global financial crisis. One of the striking aspects of the crisis is the unusually high degree of synchronization of associated recessions (and recoveries). When the last two years of the sample are taken out, the average correlation of the 2000s falls to 0.35, which is still twice as large than the average observed in the 1990s.

To emphasize the synchronicity around large recessions, Figure 15 zooms in the three-year periods around 1974 and 2008. Taking the oil embargo that started in October 1973 as the starting point, the top panel of the figure shows that there was significant divergence in the countries' experiences during this period. The U.K. is already deep in a recession while France and Canada are still expanding. By the end of 1974, when the U.S. experiences the worst point of the recession, this divergence continues, evidenced by the large range of the factors – between -1 for Japan and the U.K. and -5 for the U.S.. In great contrast, by the beginning of 2007, all countries have converged in a very narrow band and in the worst point of the Great Recession, all countries are in a recession. Excluding Germany and Japan, which are the two export-dependent countries that are by far the worst-affected countries, the factors are very close to each other both in terms of magnitudes and coherence.

These findings indicate that it is difficult to reach a conclusive result about the links between globalization and the synchronization of business cycles. On the one hand, there is a pick up in the degree of synchronization during the last decade (with or without crisis) suggesting a positive link. On the other hand, the average correlation in the 2000s (after the crisis years are eliminated) is only slightly higher than that of the 1970s suggesting that the extent of synchronization has not changed much even though there has been a substantial increase in global trade and financial linkages. These observations are consistent with the

findings reported in some of the earlier studies summarized in section 2.

5 Multi-Country Analysis

Country factors do not likely evolve in isolation. Hence, just as we obtain useful country models by allowing country indicators to depend on country factors, way may similarly want to allow country factors to depend on a *global* (in this case, G-7) factor. We now proceed to do so.¹⁴ Our methodology allows us to present a coherent analysis of interactions between the global business cycle and country-specific cycles, as it employs a well-defined hierarchical structure. As with our earlier single-country analysis, we first sketch our multi-country dynamic factor framework, and then we present a variety of empirical results.

5.1 A Hierarchical Multi-Country Model

Having extracted country-specific factors, we now turn to decomposing these factors into a common factor across the G-7 countries and idiosyncratic components. Since all country-specific factors are monthly, the extraction of the common factor will be a straightforward application of the approach in Stock and Watson (1989). In particular, for country j , the measurement equation will be

$$x_t^j = \mu^j + \theta^j f_t + \xi_t^j \quad (7)$$

where f_t is the common G-7 factor which has the transition equation

$$f_t = \omega_1 f_{t-1} + \omega_2 f_{t-2} + \omega_3 f_{t-3} + \zeta_t \quad (8)$$

and ξ_t^j is the country-specific component with the transition equation

$$\xi_t^j = \delta_1 \xi_{t-1}^j + \delta_2 \xi_{t-2}^j + \delta_3 \xi_{t-3}^j + \tau_t^j. \quad (9)$$

As is standard, we assume that ζ_t and τ_t^j are uncorrelated among each other and iid over time.¹⁵

¹⁴Although we do not pursue the possibility here due to the relatively small size and coherent nature of the G-7 countries, with larger sets of countries one might want to allow for the possibility of an intermediate layer of *regional* factors (e.g., Europe, Asia, Mideast, ...) such that country factors depend on regional factors, which depend on global factors.

¹⁵In order to satisfy identification in the factor model, we normalize the variance of ζ such that f_t has a unit variance. This requires a non-linear restriction in $\omega_1, \omega_2, \omega_3$ and the variance of ζ .

5.2 Multi-Country Empirics and Extraction of G-7 Real Activity

In this section, we first present our estimation results with respect to the G-7 factor and explain how it relates to the underlying country factors. Then we briefly describe the temporal evolution of the G-7 factor.

A number of interesting results emerge: First, the G-7 factor picks up a substantial amount of co-movement across countries. Second, the estimation of the factor with various activity indicators definitely enhances its ability to capture common macroeconomic fluctuations in G-7 economies. Third, the G-7 factor displays some of the major global economic events of the past 40 years. In particular, the factor indicates that the global recession of 2008-2009 is more severe and longer lasting than the mid-1970s recession, making it the deepest global recession of the past forty years. Fourth, the G-7 and country-specific factors play different roles at different points in time in shaping economic activity in different countries.

5.2.1 Estimation of the G-7 Factor

The estimation results of the G-7 factor are reported in Table 6. The factor loadings, reported in the first column, show that all country factors load positively and significantly on the G-7 factor. Based on the factor loadings, Germany, France and Italy play relatively more important roles in driving the G-7 factor.¹⁶

We then analyze the extent of co-movement between G-7 factor and the country factors. The second column of the table shows that the average correlation between the G-7 factor and country factors is around 0.7 suggesting a reasonably high degree of co-movement. Across countries, however, there are variations in correlations. Canada, Germany and the U.S. feature the highest correlations (around 0.7) while Italy has the lowest one (0.53). All countries display higher correlations of country factor with the G-7 factor than with the country factors of any of the other countries.¹⁷ These findings suggest that the G-7 factor picks up a substantial amount of comovement across countries.

The extent of co-movement between the G-7 factor and country factors has varied significantly over time. While periods of low correlations reflect the dominant role played by country specific developments, periods of high correlations coincide with the common shocks.

¹⁶The roots of the autoregressive process in the transition equation are 0.96 and $0.91 \pm 0.32i$.

¹⁷In addition, we find that the correlation between output and G-7 factor of a country is greater than that of other countries' output in all cases except the pairs of Germany-France and Germany-the U.K. (19 out of 21 pair-wise correlations).

For instance, the correlation between the G-7 factor and the country factor of Japan is -0.14 (not reported) during the 1990s reflecting the country-specific nature of Japan's lost decade. In the 2000s, the correlation between the two factors is 0.87 because of the synchronized recessions of the early 2000s and 2008-2009.

In order to further examine the links between the (filtered) G-7 factor and the underlying country factors, we regress the G-7 factor on current and 12 lags of each country factors. We report the coefficient on the contemporaneous country factors and the sum of the coefficients for each variable to derive its total effect in the last two columns of Table 6. The country factors for Canada and Germany, followed by Japan and France have the largest contemporaneous effect. However, in terms of total effects the United States dominates the other countries, followed by Germany and the United Kingdom. France and Japan have very small influence in terms of the total effect.

The G-7 factor features a high level of persistence, as evidenced from the roots of the characteristic polynomials. The results indicate that the G-7 factor is slightly more persistent than the country factors.¹⁸ This suggests that the G-7 factor is able to capture low frequency (more persistent) comovement across countries. There is also evidence that the volatility of the G-7 factor has fallen over time up until the 2008-2009 recession. This is consistent with the evolution of the volatility of country factors we analyzed in the previous section.

5.2.2 Evolution of the G-7 Factor and the Idiosyncratic Components

The G-7 factor extracted from the country factors is shown in Figure 16. It is able to capture some of the major economic events of the past 40 years. In particular, the behavior of the G-7 factor is consistent with the recession of the mid-1970s (associated with the first oil price shock), the recession of the early 1980s (associated with the tight monetary policies of major industrialized nations), the expansionary period of the late 80s, the recession of the early 1990s, the expansionary period of the late 1990s, the downturn of 2001, and the global recession of 2008-2009, and the subsequent recovery. The depth and highly synchronized nature of the mid-1970s and the latest recession episode is striking. However, the G-7 factor shows that the latest episode is much more severe and longer lasting than the mid-1970s recession, making it the deepest global recession of the past forty years.¹⁹ Indeed, according

¹⁸This result is similar to those reported by Gregory et al. (1997); and Kose et al. (2003a).

¹⁹The major recession dates identified by the G-7 factor are consistent with the dates of the global recessions documented by Kose et al. (2009) and Kose et al. (2010). Using a sample of more than 190 countries, they also conclude that the 2009 recession is the deepest and most synchronized global recession episode of the post-war era. Imbs (2009), using monthly data on industrial production to study the evolution

to our results, the latest episode resulted in a decline in economic activity by 1.8 times larger and lasted twice as long as the mid-1970s recession.

The lower panels of Figure 8-14 show the decomposition of each country factor (shown in the upper panel) in to a part that is due to the G-7 factor and an idiosyncratic part. Using the notation (7), the former is $\theta^j f_t$ and the latter is ξ_t^j , for a country j . We have already reported that the extent of co-movement between the G-7 factor and county factors is quite high, but it does vary over time. This simply reflects that while the G-7 factor is tightly related to the country factors, there are periods where they move in different directions due to country-specific (idiosyncratic) developments. For example, in the case of Japan, the idiosyncratic factor confirms that the “Lost Decade” is clearly specific to Japan (Figure 12). The idiosyncratic factor also displays that the amplitude of the mid-1970s recession in Japan is larger because of Japan-specific reasons, i.e., its heavy reliance on imported oil. Although the G-7 factor reflects the synchronized nature of the 2008-2009 recession, the sharp drop in the idiosyncratic factor suggests that there are country-specific forces making the Japan’s recession deeper during this period.

In the case of Canada, the G-7 factor contributes to the severity of the mid-1970s recession while the idiosyncratic factor plays a mitigating role (Figure 8). In contrast, the idiosyncratic factor aggravates the impact of the early 1980s and early 1990s recessions in Canada suggesting that country-specific developments play a substantial role during those episodes. The severe recession of 2008-2009 is mostly driven by the G-7 factor while the idiosyncratic factor plays a mitigating role in Canada.

These results suggest that the G-7 and country-specific factors play different roles at different points in time in different countries. In some periods, the country factor is more strongly reflective of domestic activity, while in others the domestic activity reflects the movements in the G-7 factor.

6 Concluding Remarks

In this paper we hope to have cast new light on the global business cycle and its evolution, working in the tradition of Gregory et al. (1997) and Kose et al. (2003a), among others. Those authors, however, use only a few low-frequency quarterly indicators from the national income and product accounts. In contrast, we use more indicators when possible (and higher-

of business cycle correlations since the 1980s, concludes that the degree of cross-country business cycle correlations during the latest crisis is the highest in three decades.

frequency, monthly). We do so because lurking in the background is our hope to push toward *real-time* monitoring of global macroeconomic activity, which we have not emphasized thus far but now wish to highlight.

The past quarter-century has witnessed not only progress toward globalization, but also unprecedented progress in information technology (collection, transmission, processing, storage, etc.), and one wants to monitor the global business cycle in ways that exploit the IT revolution. In part we are doing so already, exploiting, for example, newly-available monthly data. But more could be done using IT advances in global real-time monitoring, building for example on the single-country “nowcasting” framework of Aruoba et al. (2009) and Aruoba and Diebold (2010).²⁰ We look forward pursuing this in future work, which we hope will ultimately help guide global real-time policy formulation, implementation and analysis in global environments, in the spirit of the inaugural Feldstein Lecture to the NBER given by Taylor (2009).

²⁰Note that in such an environment, updates of assessed global activity will occur much more often than monthly, even if monthly is the highest-frequency data used, as different indicators (and revisions) are released at different times for different indicators and countries.

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Table 1: G-7 Real Activity Indicators

U.S. Series	Source	Range
Payroll Employment	Bureau of Haver (LANAGRA@USECON)	1970M1-2009M12
Gross Domestic Product	Haver (GDPH@USNA)	1970Q1-2009Q4
Household Disposable Income	Datastream (USOCFRDID)	1970Q1-2009Q4
Initial Unemp. Claims	Haver (LICM@USECON)	1970M1-2009M12
Industrial Prodn. Index	FRED (INDPRO)	1970M1-2009M12
Total Retail Trade Index	OECD (111.SLRTTO01.IXOBSA)	1970M1-2009M12
Canada Series	Source	Range
Civilian Emp., All Persons	OECD (156.EMESCVTT.STSA)	1970M1-2009M12
Gross Domestic Product	Haver (S156NGPC@G10)	1970Q1-2009Q4
Household Disposable Income	Datastream (CNOCFRDID)	1970Q1-2009Q4
Initial Unemp. Claims	Haver (V383900@CANSIM)	1970M1-2009M12
Industrial Prodn. Index	Datastream (CNI66..CE)	1970M1-2009M12
Total Retail Trade Index	OECD (156.SLRTTO01.IXOBSA)	1970M1-2009M12
U.K. Series	Source	Range
Total Employment, All Persons	OECD (112.EMESCVTT.STSA)	1971M2-2009M11
Gross Domestic Product	Haver (ABMIQ@UK)	1970Q1-2009Q4
Household Disposable Income	Datastream (UKI66..CE)	1970Q1-2009Q4
Industrial Prodn. Index	Datastream (UKI66..CE)	1970M1-2009M12
Total Retail Trade Index	OECD (112.SLRTTO01.IXOBSA)	1970M1-2009M12
Japan Series	Source	Range
Labor Force Survey: Total Emp.	Haver (FLED2@JAPAN)	1970M1-2009M12
Gross Domestic Product	OECD (158.GDPV)	1970Q1-1979Q4
	Haver (REDPC2@JAPAN)	1980Q1-2009Q4
Household Disposable Income	Datastream (JPOCFRDID)	1970Q1-2009Q4
Empl. Ins.: Initial Beneficiaries	Haver (EIIB@JAPAN)	1976M1-2009M12
Industrial Prodn. Index	Datastream (JPI66..CE)	1970M1-2009M12
Total Retail Trade Index	OECD (158.SLRTTO01.IXOBSA)	1970M1-2009M12
France Series	Source	Range
Total Employment	OECD (132.ET)	1970Q1-2009Q4
Gross Domestic Product	Haver (FRSNGDPC@FRANCE)	1970Q1-2009Q4
Gross Household Disp. Income	OECD (132.YDRH.G)	1970Q1-1977Q4
	Datasream (FROCFRDID)	1978Q1-2009Q4
Init. Claims Index, All Persons	IMF.stat	1986:M1-2009:M1
Industrial Prodn. Index	Datastream (FRI66..CE)	1970M1-2009M12
Germany Series	Source	Range
Employment	Haver (S134ELE@G10)	1981M1-2009M12
Gross Domestic Product	GDS (C134NGDP_R)	1970Q1-2009Q4
Gross Household Disp. Income	Datastream (BDPERDISP)	1970Q1-1990Q4
	OECD (134.YDRH.G)	1991Q1-2009Q4
Industrial Prodn. Index	Datastream (BDI66..CE)	1970M1-2009M12
Italy Series	Source	Range
Total Employment	OECD (136.ET)	1970Q1-2009Q4
Gross Domestic Product	OECD (136.GDPV)	1970Q1-1980Q4
	Haver (ITSNGDPC@ITALY)	1981Q1-2009Q4
Gross Household Income	OECD (136.YDRH.G)	1961Q1- 2008Q4
Industrial Prodn. Index	Datastream (ITI66..CE)	1970:M1-2009:M12

Note: The table reports the series names, sources (with source-specific mnemonics in parentheses) and ranges for the variables used.

Table 2: G-7 Country Estimation Results

Indicator	U.S.	Canada	U.K.	Japan	France	Germany	Italy
EMP	1.86 (*)	1.98 (*)	0.26 (*) 1.15 (*)	0.69 (*)	0.15 (*)	0.47 (*)	0.28 (*)
IP	7.05 (*)	4.57 (*)	5.24 (*)	8.69 (*)	5.42 (*)	7.26 (*)	12.34 (*)
RET	4.18 (*)	2.46 (*)	2.05 (*)	2.95(*)	—	—	19.96 (*)
INC	0.56 (*)	0.58 (*)	0.68 (*)	0.71 (*)	0.33 (*)	0.49 (*)	0.23
GDP	1.06 (*)	0.92 (*)	1.27 (*)	1.24 (*)	0.83 (*)	1.11 (*)	1.87 (*)
INIT	-0.01 (*)	-0.11 (*)	—	0.00	0.00	—	—
ρ_1	0.64 (*)	2.27 (*)	0.48 (*)	1.95 (*)	1.54 (*)	1.60 (*)	-0.09 (*)
ρ_2	-0.05	-1.90 (*)	0.27 (*)	-1.57 (*)	-1.14 (*)	-1.09 (*)	0.27 (*)
ρ_3	0.18 (*)	0.60 (*)	-0.10	0.56 (*)	0.50 (*)	0.33 (*)	0.45 (*)
λ_1	0.84	0.89	0.66	0.90	0.90	0.74	0.85
λ_2	-0.10 + 0.45i	0.69 + 0.44i	-0.49	0.53+0.59i	0.32+0.68i	0.43+0.52i	-0.47+0.55i
λ_3	-0.10 - 0.45i	0.69 - 0.44i	0.31	0.53-0.59i	0.32-0.68i	0.43-0.52i	-0.47-0.55i

Note: The variable acronyms correspond to the measures of employment, industrial production, retail sales, income, GDP and initial claims, as defined in Table 1. For the U.K., employment before and after 1992 appear as two separate variables. In the top panel we show, for each country, the estimated indicator loadings on the country factor. In the middle panel we show the estimated transition equation dynamic parameters. In both the top and middle panels, asterisks indicate statistical significance at the five percent level. (*) denotes significance at 5% level. In the bottom panel we show the inverted roots, sorted from largest to smallest modulus, corresponding to the estimated transition equation dynamic parameters.

Table 3: Country Factors and Indicators

(a) Correlations Between Smoothed Country Factors and Indicators

Indicator	U.S.	Canada	U.K.	Japan	France	Germany	Italy
EMP	0.87	0.60	0.50 0.77	0.20	0.20	0.43	0.09
GDP	0.63	0.82	0.76	0.80	0.89	0.61	0.77
INC	0.32	0.35	0.26	0.47	0.36	0.33	0.33
INIT	-0.35	-0.16	-	-0.29	-0.07	-	-
IP	0.83	0.41	0.43	0.48	0.37	0.43	0.62
RET	0.32	0.16	0.15	0.21	-	-	0.57

(b) Weights of Contemporaneous Indicators in the Filtered Country Factors

Indicator	U.S.	Canada	U.K.	Japan	France	Germany	Italy
EMP	0.55	0.19	0.43	0.09	0.02	0.16	0.00
IP	0.32	0.11	0.19	0.18	0.13	0.32	0.20
RET	0.06	0.04	0.04	0.08	-	-	0.23
INIT	-0.46	-0.43	-	0.10	-0.05	-	-
INC	0.04	0.05	0.05	0.11	0.02	0.09	0.04
GDP	0.21	0.34	0.58	0.53	0.80	0.45	0.62

(c) Cumulative Weights of Indicators in the Filtered Country Factors

Indicator	U.S.	Canada	U.K.	Japan	France	Germany	Italy
EMP	0.13	0.49	0.01	0.20	0.00	0.09	0.01
IP	0.56	0.31	0.34	0.24	0.08	0.38	-0.13
RET	0.14	0.13	0.21	0.14	-	-	-0.22
INIT	-0.01	-0.02	-	0.01	-0.01	-	-
INC	0.06	0.06	0.03	0.09	0.00	0.05	0.00
GDP	0.36	0.55	0.76	0.74	0.87	0.74	0.76

Note: The variable acronyms correspond to the measures of employment, industrial production, retail sales, income, GDP and initial claims, as defined in Table 1. For the U.K., employment before and after 1992 appear as two separate variables. In panel (a) we show, for each country, the contemporaneous correlations between indicators and the smoothed country factors. For panels (b) and (c), we regress the filtered country factors on current and 12 monthly or 4 quarterly lags of standardized indicators using only the end-of-quarter observations. Panel (b) reports the coefficient on the contemporaneous indicators and panel (c) reports the sum of the coefficients for each indicator.

Table 4: Months with Lowest Observations for Each Country Factor

US	UK	CAN	JAP	FRA	GER	ITA
<i>1974M12</i>	2009M01	1982M05	2008M12	2008M11	2009M01	2009M01
1980M05	2008M11	1982M06	2009M01	<i>1974M11</i>	2008M12	2008M11
<i>1974M11</i>	2009M03	1982M04	2008M11	2008M12	2008M11	<i>1974M10</i>
<i>1975M01</i>	2009M02	1982M07	2009M02	2009M01	2009M02	<i>1974M12</i>
1980M06	1980M02	2009M02	2008M10	<i>1974M12</i>	2008M10	1992M08
2008M12	1973M11	2009M03	<i>1974M02</i>	2008M10	2009M03	2009M02
2008M11	<i>1975M04</i>	1982M03	<i>1974M01</i>	2009M02	<i>1974M12</i>	2008M12
2009M01	<i>1974M02</i>	2009M01	<i>1974M03</i>	<i>1974M10</i>	<i>1974M11</i>	2009M03
1970M10	2008M12	1982M08	2008M09	<i>1975M01</i>	<i>1975M01</i>	2008M10
<i>1975M02</i>	1980M04	1990M12	1998M03	2009M03	<i>1974M10</i>	<i>1975M01</i>

Note: The table reports the month with the lowest observations for each country, in ascending order. Boldface denotes months in 2008 and 2009 and italics denote the months in 1974 and 1975.

Table 5: Evolution of Correlation of Country Factors

Full Sample [Average = 0.44]							1990s [Average = 0.19]					
	CAN	FRA	GER	ITA	JAP	UK	CAN	FRA	GER	ITA	JAP	UK
FRA	0.47						0.49					
GER	0.42	0.64					-0.07	0.46				
ITA	0.37	0.55	0.42				0.22	0.41	0.17			
JAP	0.31	0.48	0.59	0.37			-0.42	-0.22	0.32	0.04		
UK	0.45	0.40	0.48	0.25	0.37		0.74	0.38	0.04	0.21	-0.28	
US	0.60	0.43	0.49	0.29	0.41	0.43	0.77	0.35	-0.05	0.18	-0.33	0.57
1970s [Average = 0.37]							2000s [Average = 0.74]					
	CAN	FRA	GER	ITA	JAP	UK	CAN	FRA	GER	ITA	JAP	UK
FRA	0.48						0.81					
GER	0.42	0.52					0.68	0.84				
ITA	0.24	0.51	0.27				0.66	0.70	0.68			
JAP	0.16	0.27	0.54	0.12			0.69	0.83	0.84	0.66		
UK	0.43	0.33	0.57	0.08	0.52		0.78	0.73	0.67	0.64	0.69	
US	0.38	0.44	0.53	0.16	0.46	0.33	0.83	0.83	0.70	0.64	0.80	0.73
1980s [Average = 0.33]							2000s (excl. 08-09) [Average = 0.35]					
	CAN	FRA	GER	ITA	JAP	UK	CAN	FRA	GER	ITA	JAP	UK
FRA	-0.03						0.59					
GER	0.47	0.61					0.25	0.60				
ITA	0.28	0.27	0.31				0.32	0.36	0.32			
JAP	0.42	0.40	0.34	0.23			0.48	0.54	0.51	0.31		
UK	0.17	0.30	0.45	0.22	0.20		0.14	0.20	0.05	0.11	0.23	
US	0.70	0.10	0.51	0.23	0.37	0.29	0.55	0.51	0.21	0.22	0.59	0.20

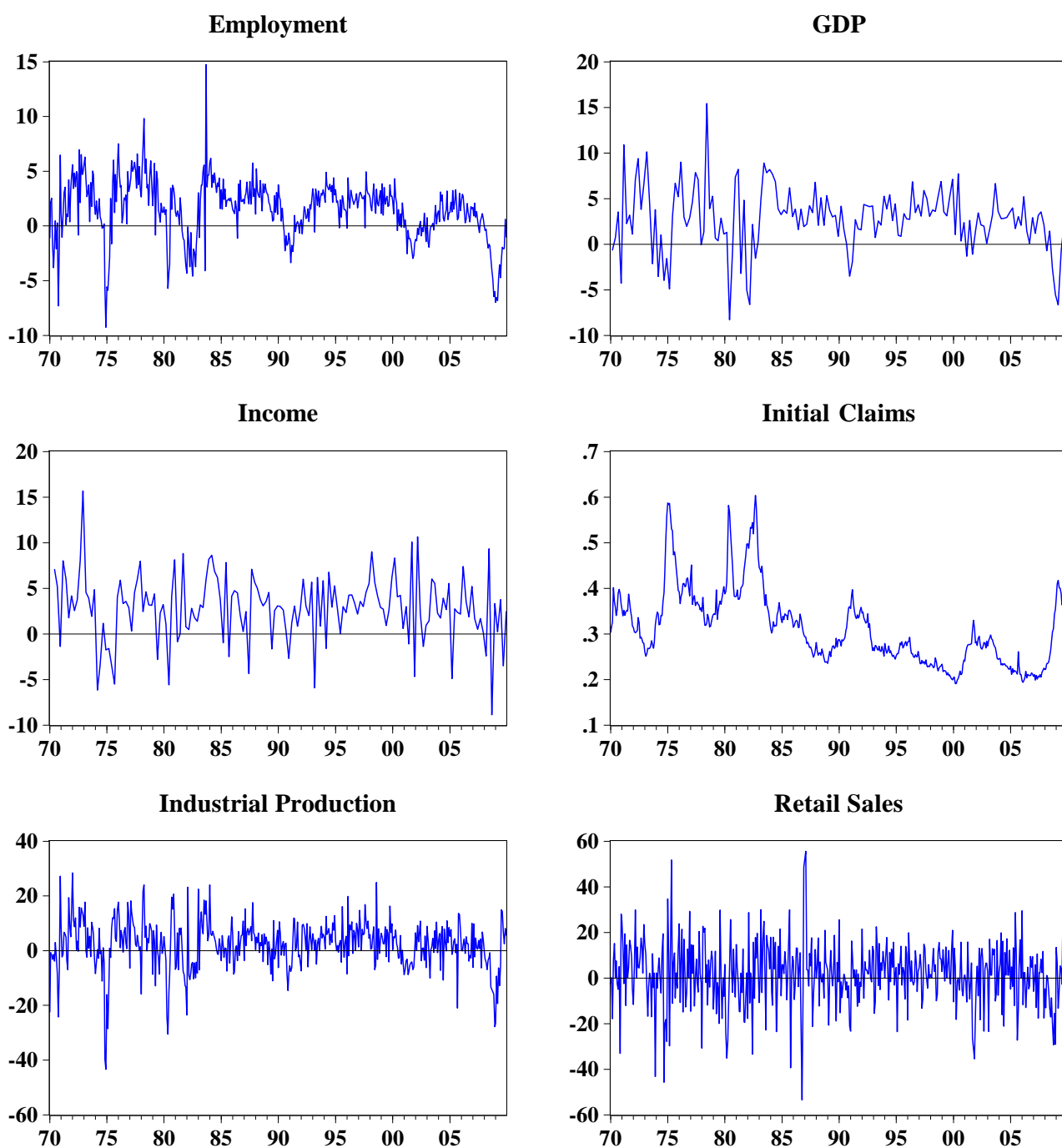
Note: The table reports the pairwise correlations between the country factors.

Table 6: Estimation Results for the G-7 Factor

	Factor Loadings	Contemporaneous Correlation with G-7 Factor	Weights of Country Factors	
			Contemporaneous	Cumulative
Canada	0.62	0.76	0.74	0.19
France	0.61	0.71	0.28	0.09
Germany	0.70	0.75	0.51	0.27
Italy	0.41	0.53	0.05	0.14
Japan	0.49	0.60	0.28	0.05
U.K.	0.55	0.63	0.07	0.26
U.S.	0.68	0.74	0.10	0.34

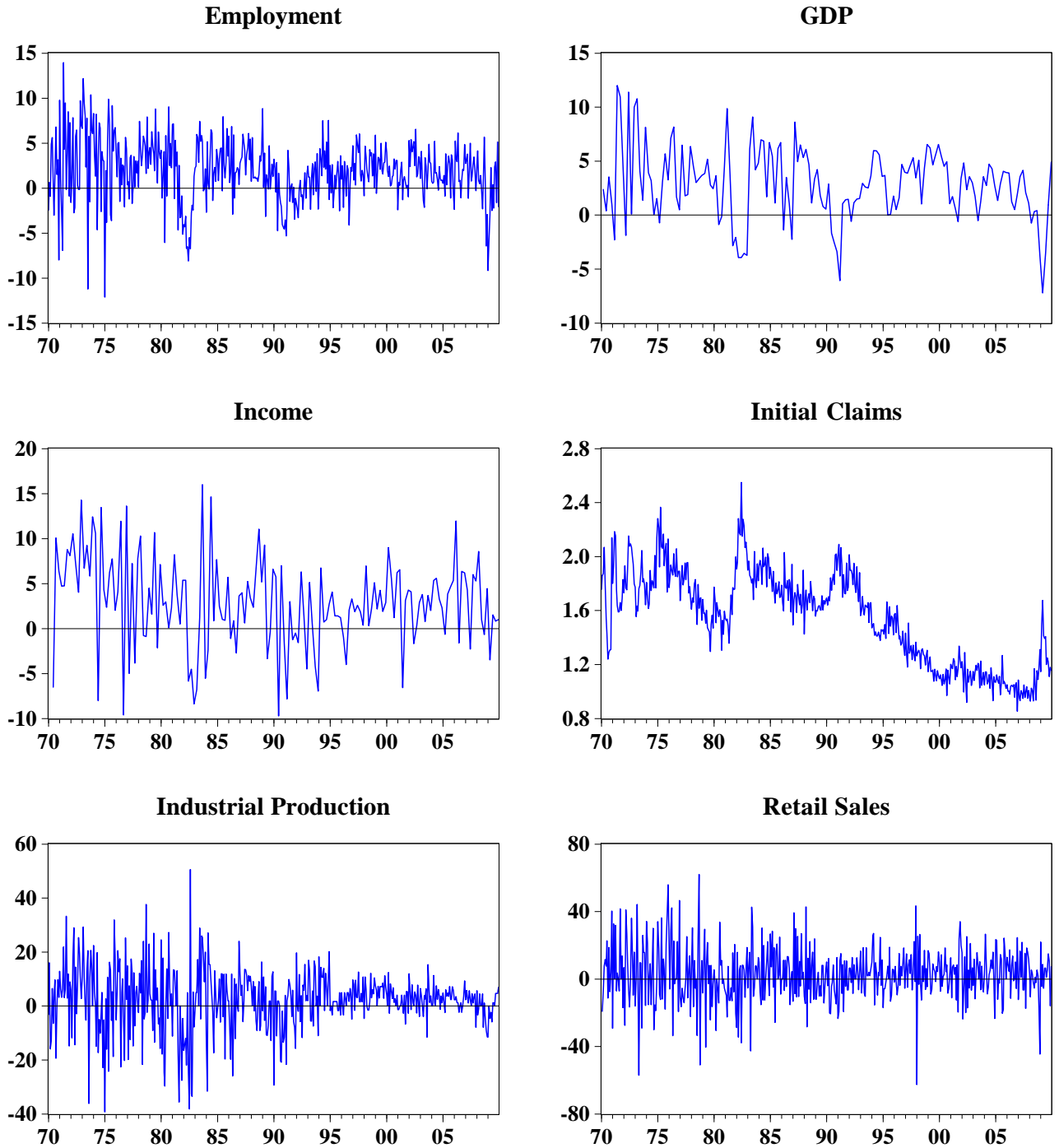
Note: The first column reports the estimate G-7 factor loadings for each country factor. All estimates are highly significant. The second column shows the correlation between the smoothed G-7 factor and each country factor. For the last two columns, we regress the filtered G-7 factor on current and 12 monthly lags of standardized country factors. Panel (b) reports the coefficient on the contemporaneous country factors and panel (c) reports the sum of the coefficients for each country.

Figure 1: U.S. Real Activity Indicators



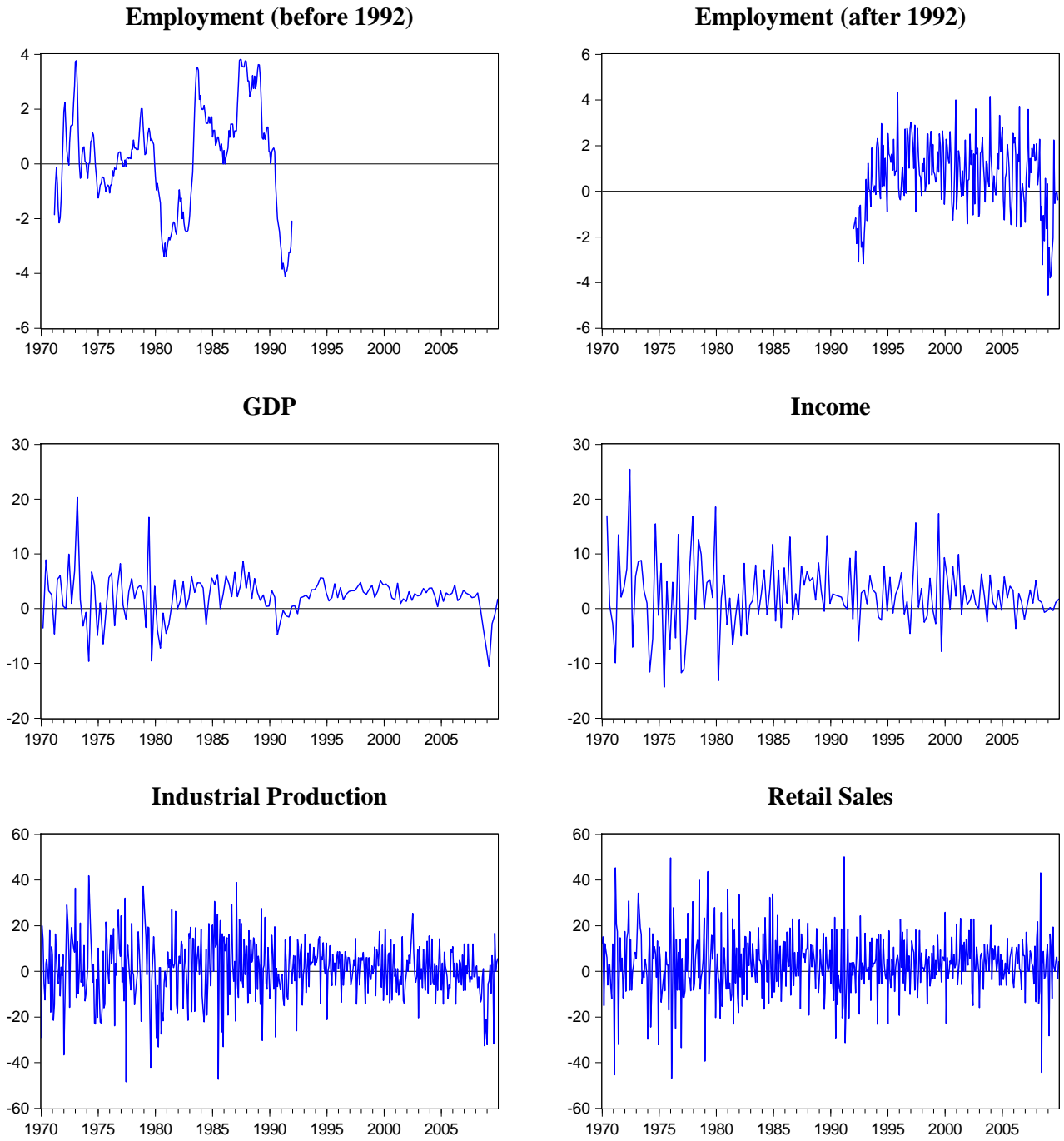
Note: All variables except Initial Claims are expressed as annualized monthly or quarterly percentage changes. Initial Claims is expressed as a percentage of the total labor force.

Figure 2: Canada Real Activity Indicators



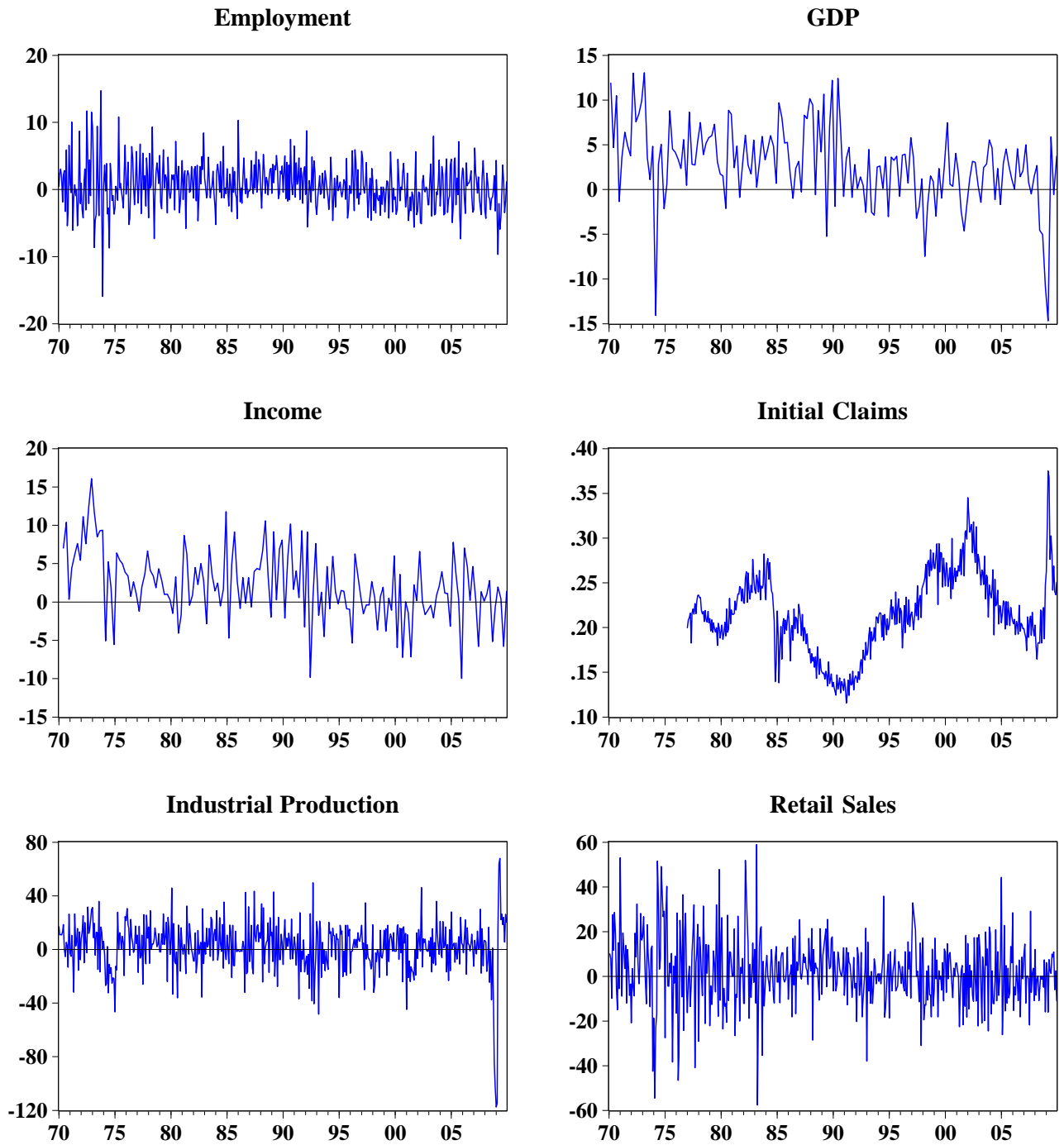
Note: All variables except Initial Claims are expressed as annualized monthly or quarterly percentage changes. Initial Claims is expressed as a percentage of the total labor force.

Figure 3: U.K. Real Activity Indicators



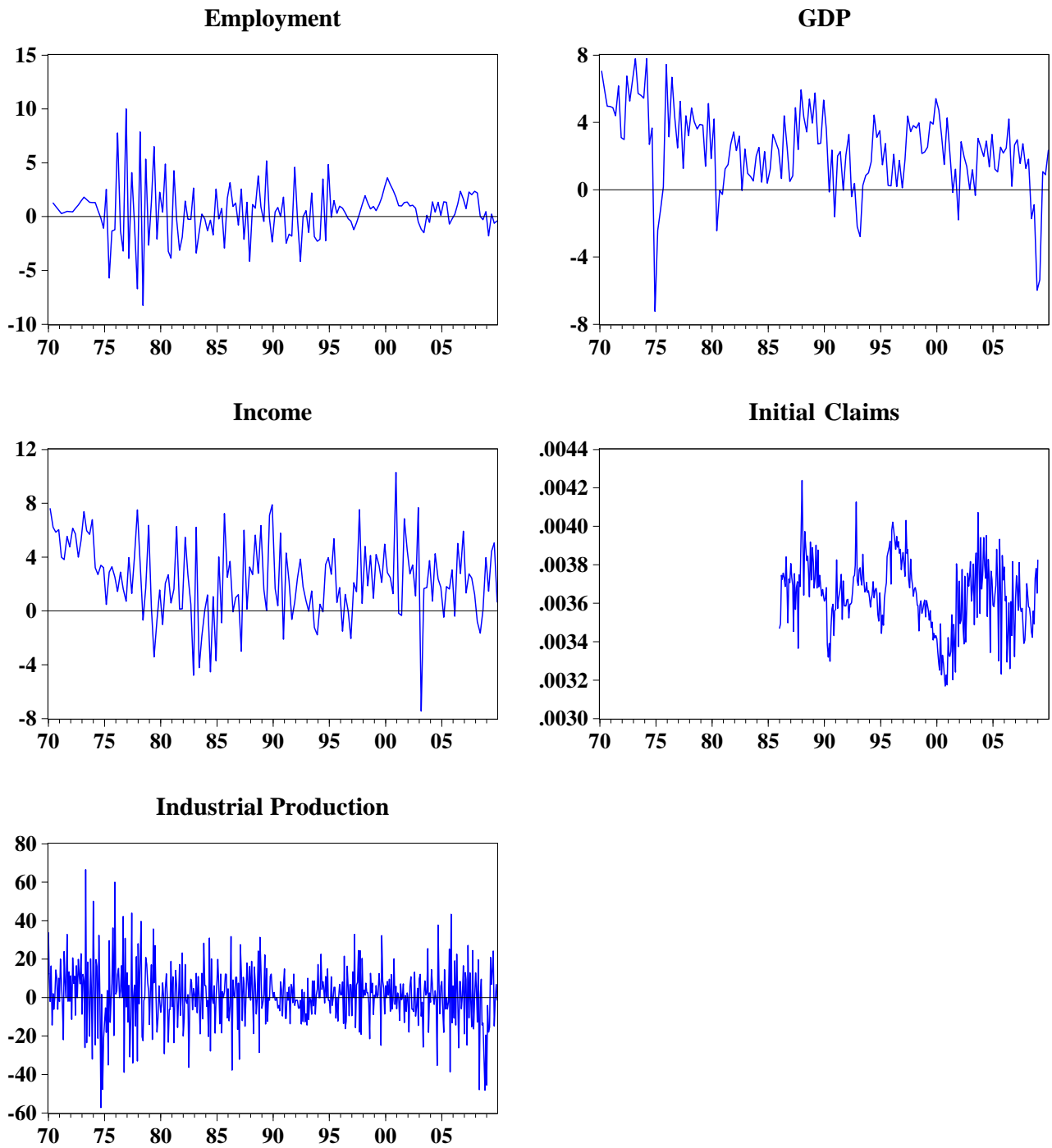
Note: All variables are expressed as annualized monthly or quarterly percentage changes. Employment is treated as two separate series before and after 1992.

Figure 4: Japan Real Activity Indicators



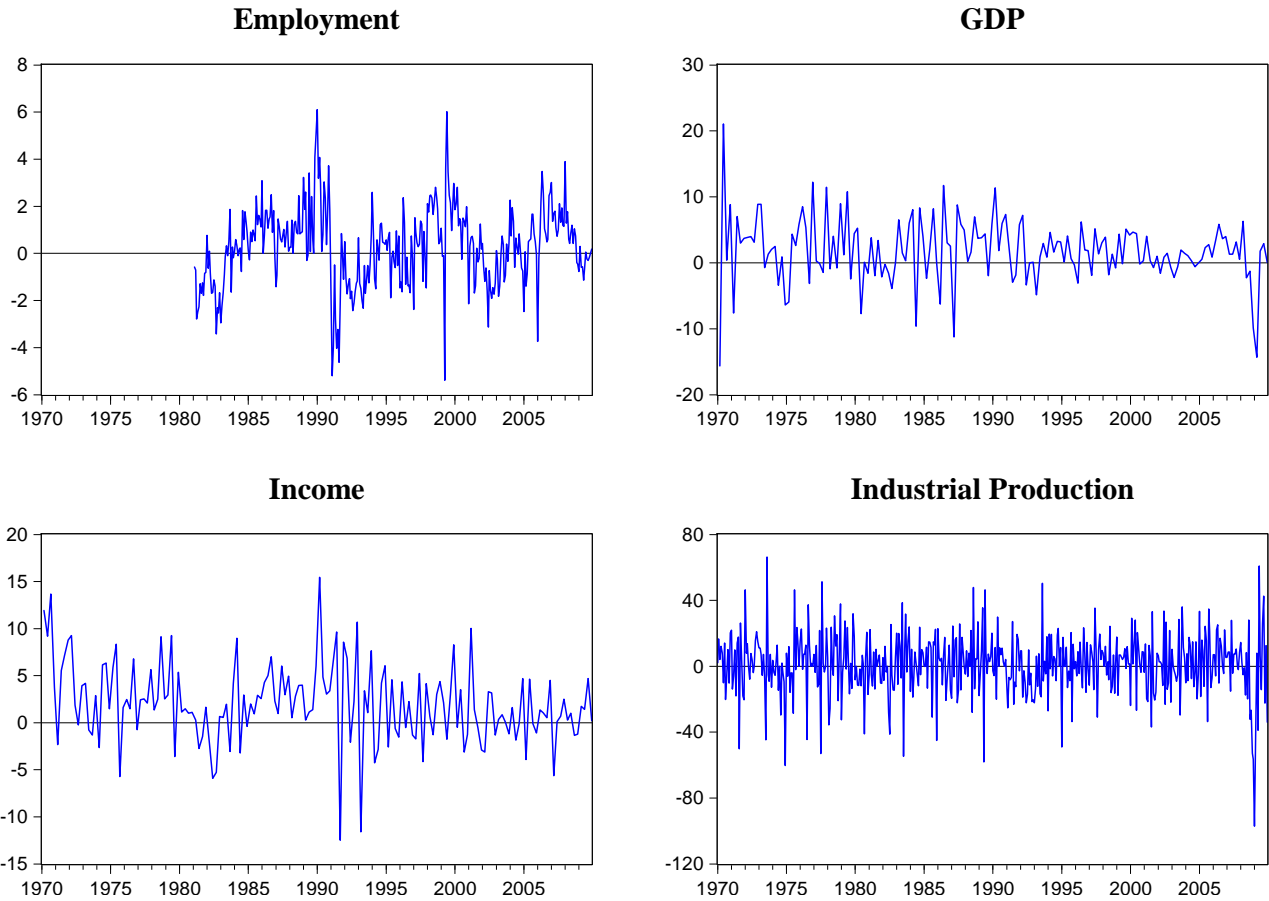
Note: All variables except Initial Claims are expressed as annualized monthly or quarterly percentage changes. Initial Claims is expressed as a percentage of the total labor force.

Figure 5: France Real Activity Indicators



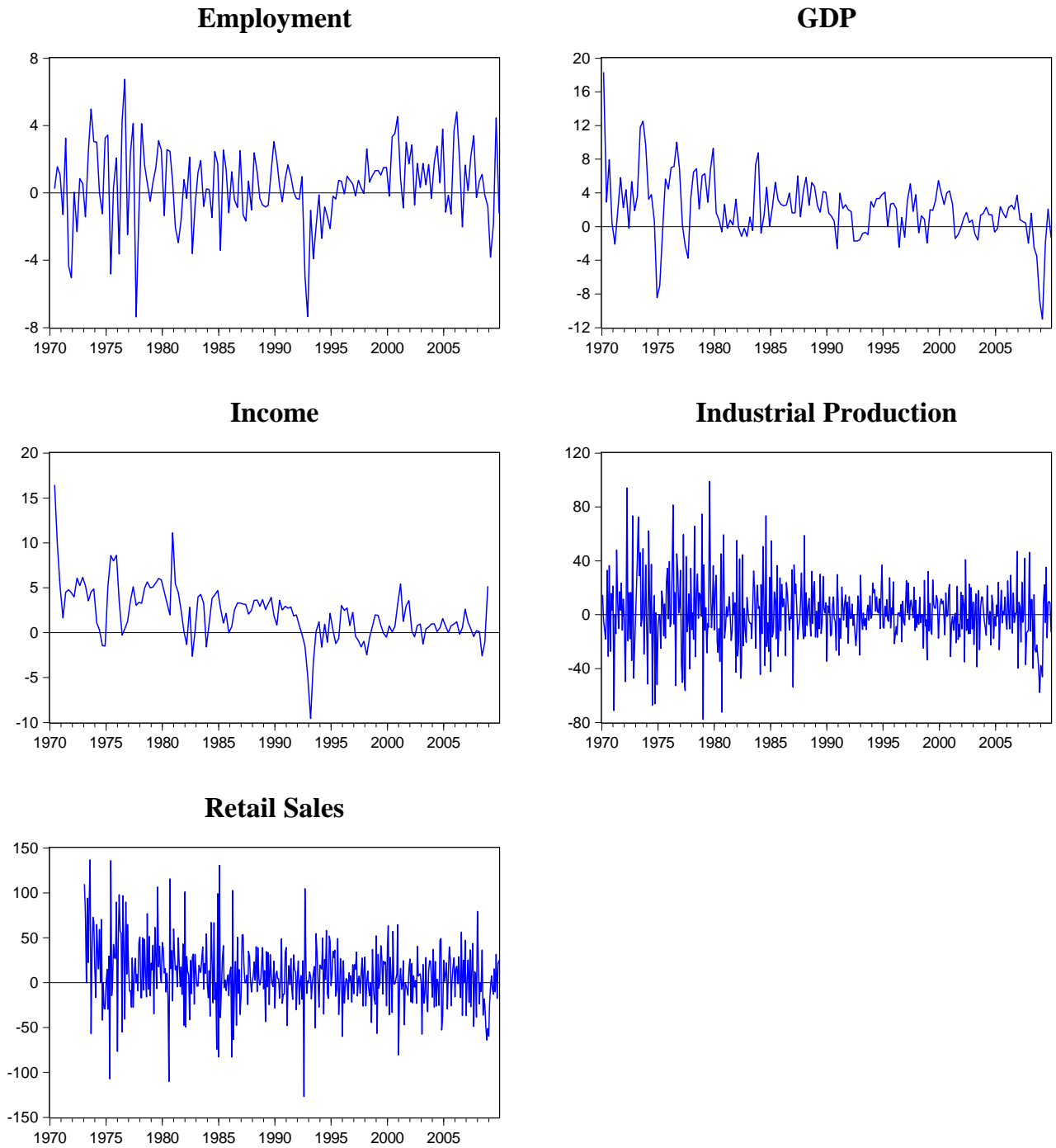
Note: All variables except Initial Claims are expressed as annualized monthly or quarterly percentage changes. Initial Claims is expressed relative to the total labor force but is in arbitrary units since it is an index.

Figure 6: Germany Real Activity Indicators



Note: All variables are expressed as annualized monthly or quarterly percentage changes.

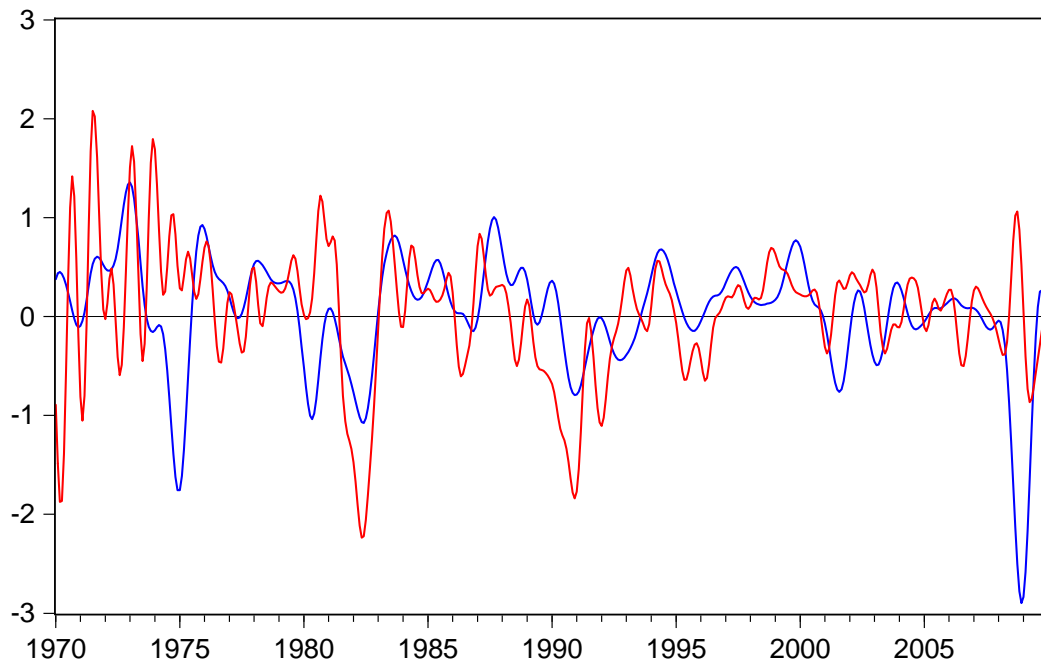
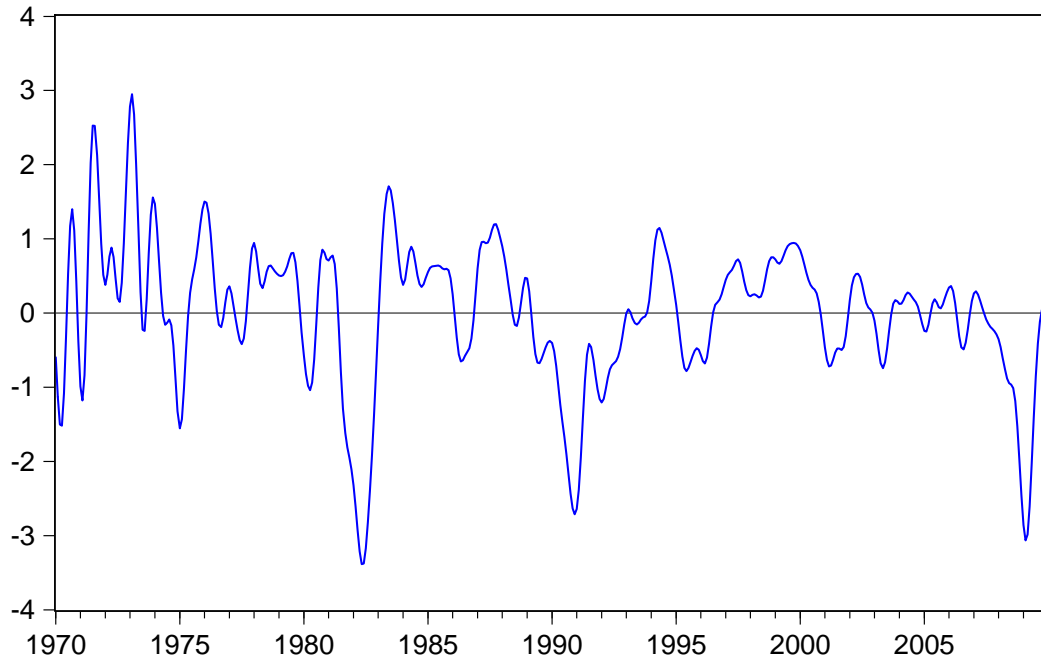
Figure 7: Italy Real Activity Indicators



Note: All variables are expressed as annualized monthly or quarterly percentage changes.

Figure 8: Country Factors : Canada

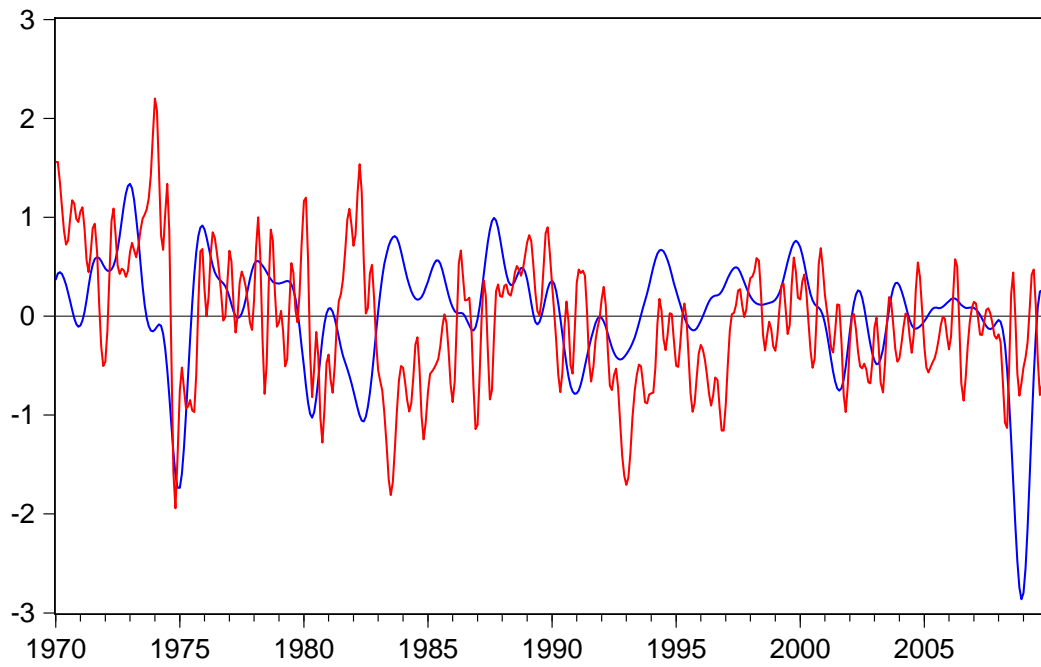
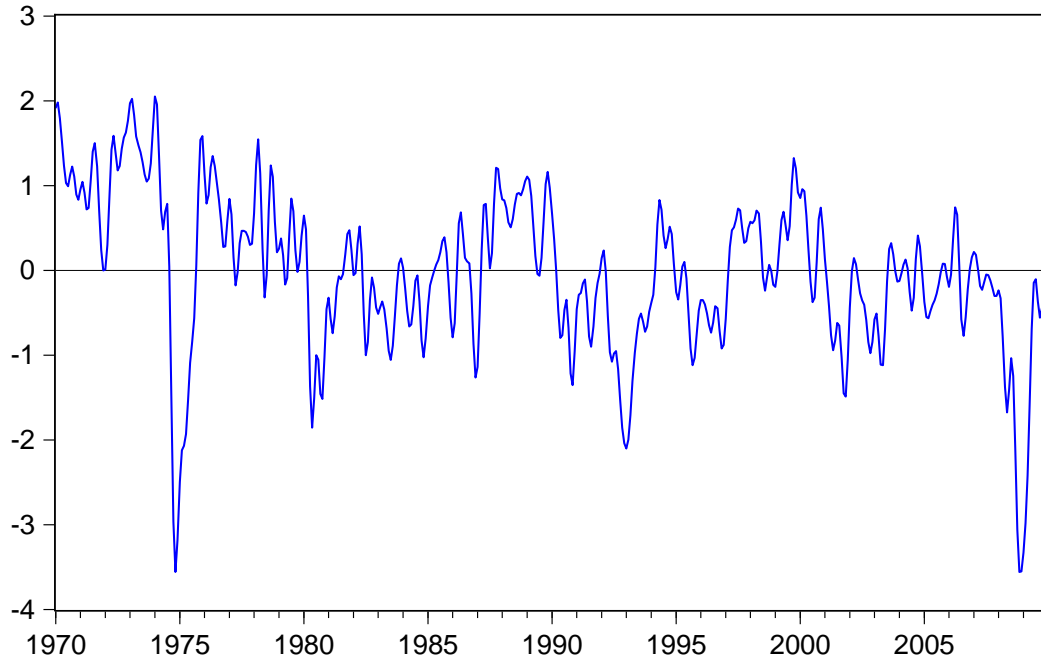
Smoothed Factor



— Common (G-7) Component
— Idiosyncratic Component

Figure 9: Country Factors : France

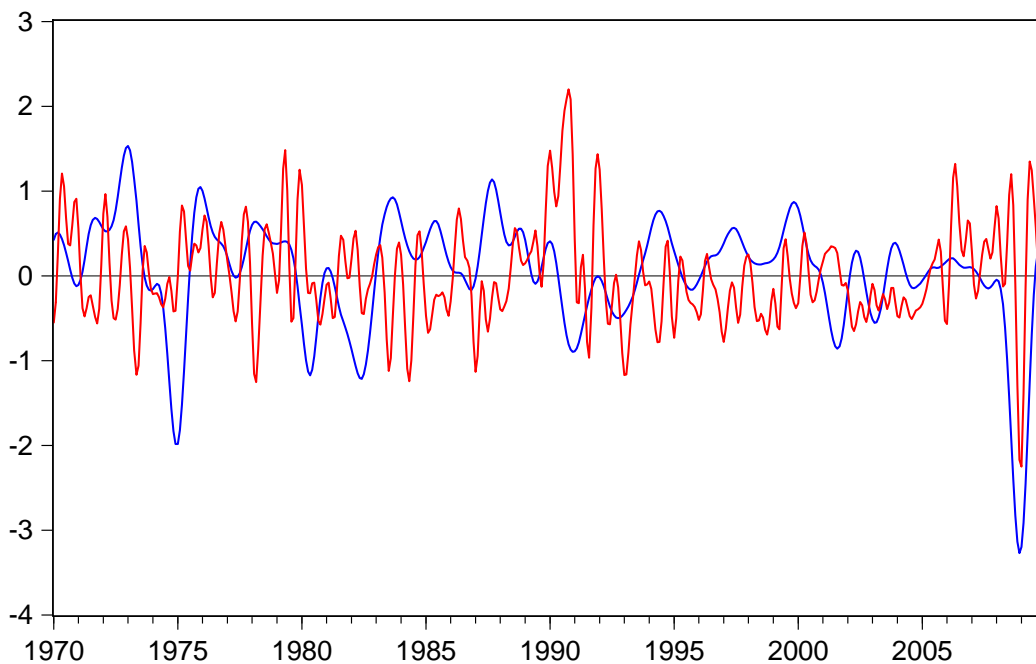
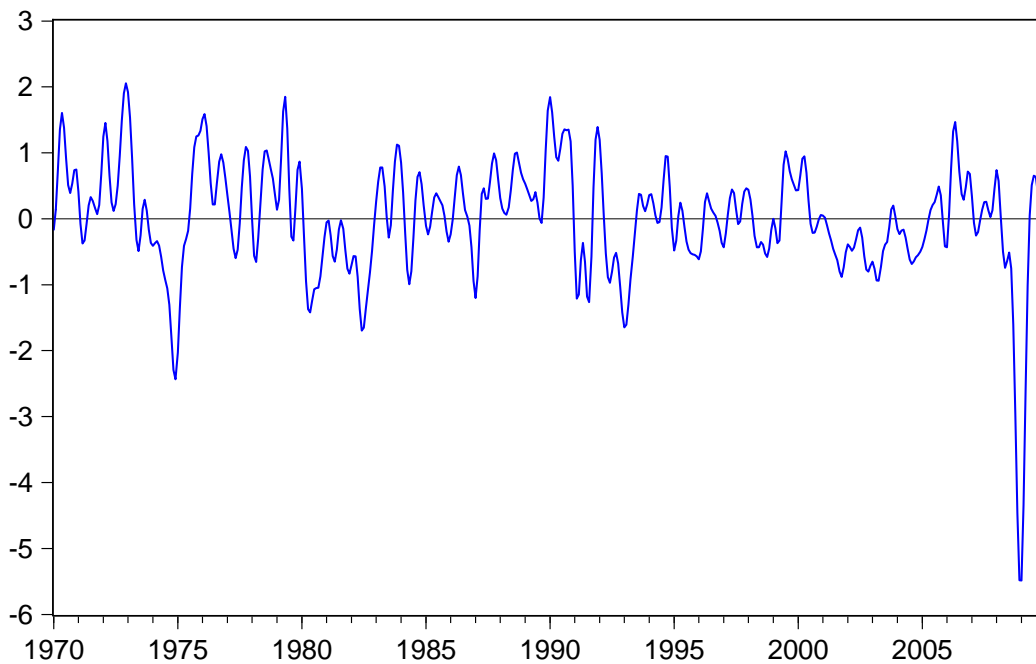
Smoothed Factor



— Common (G-7) Component
— Idiosyncratic Component

Figure 10: Country Factors : Germany

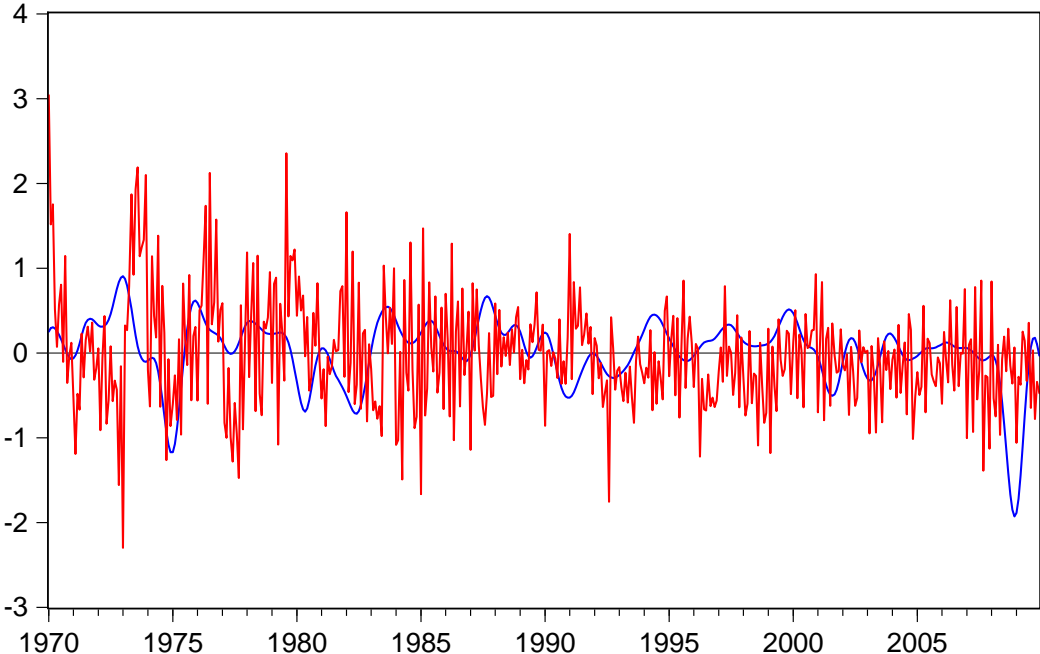
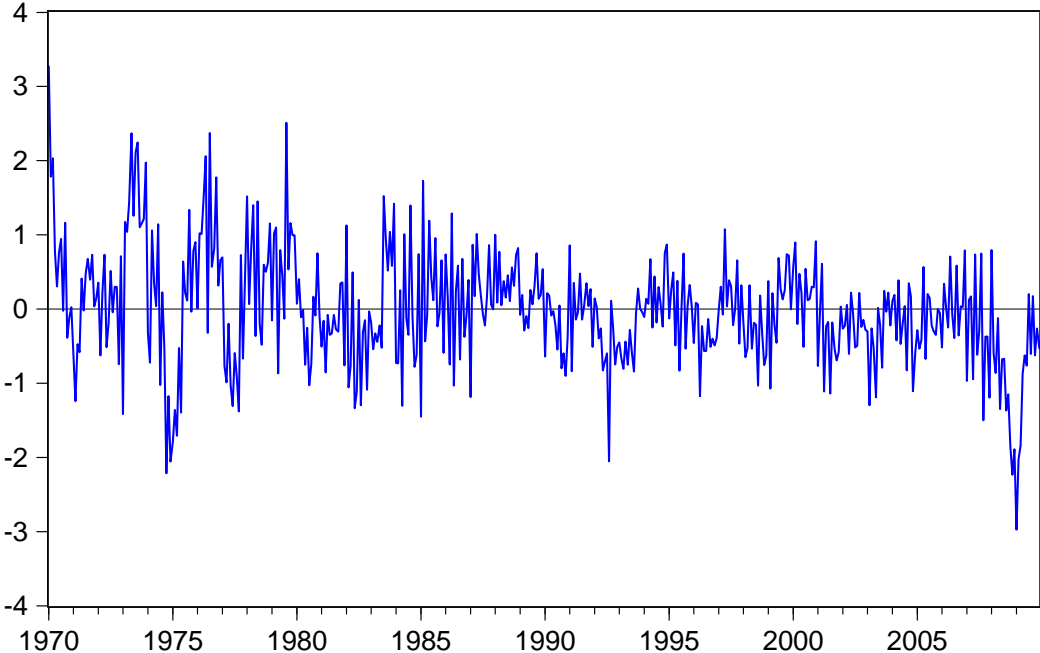
Smoothed Factor



— Common (G-7) Component
— Idiosyncratic Component

Figure 11: Country Factors : Italy

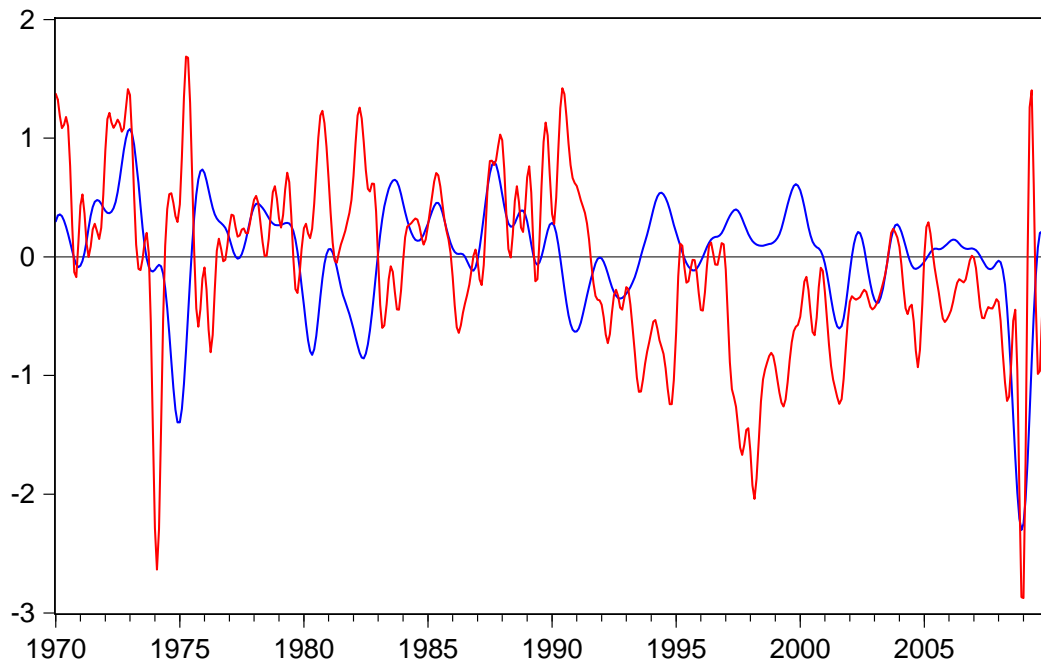
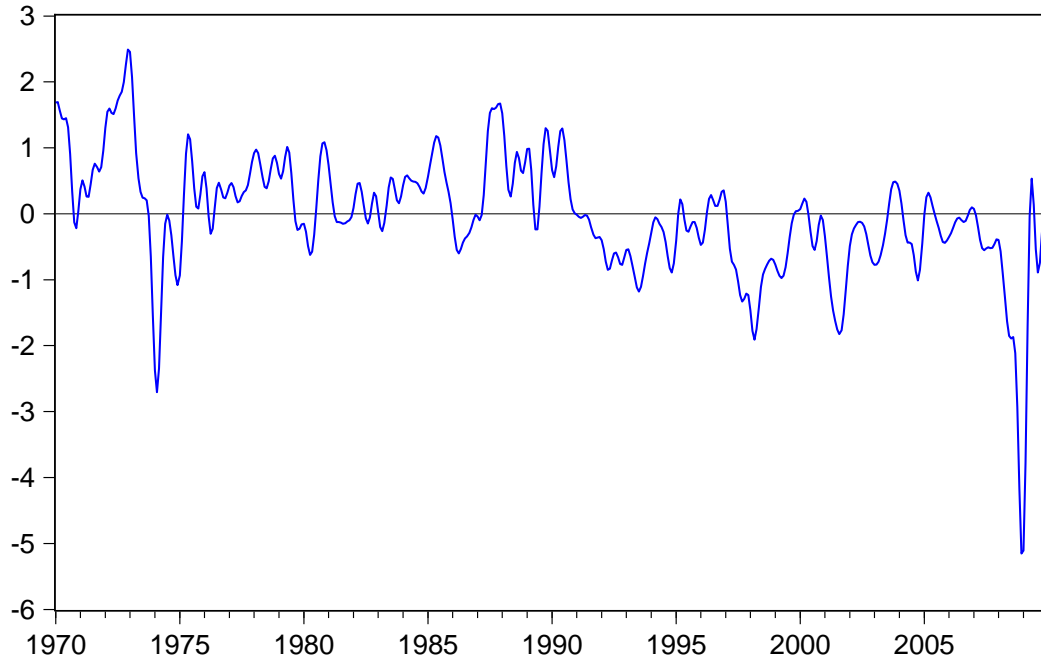
Smoothed Factor



— Common (G-7) Component
— Idiosyncratic Component

Figure 12: Country Factors : Japan

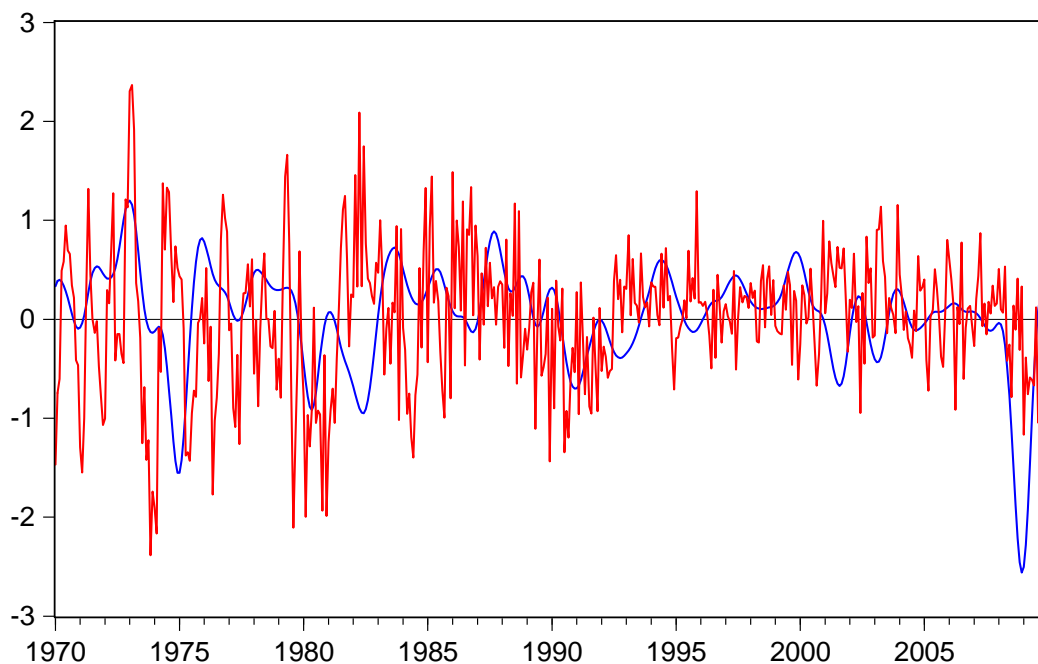
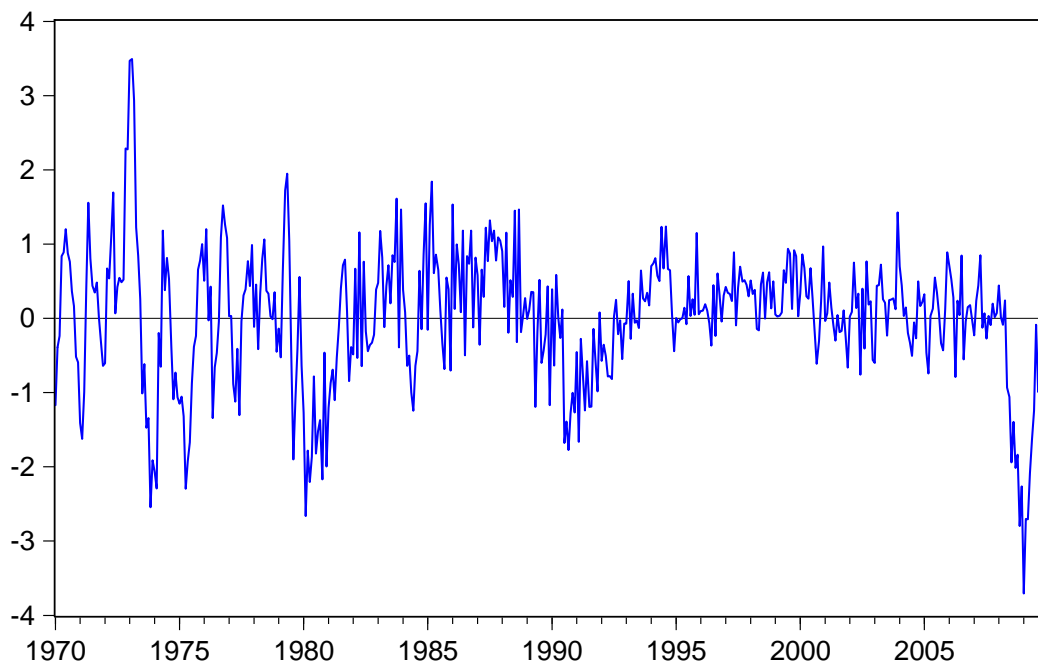
Smoothed Factor



— Common (G-7) Component
— Idiosyncratic Component

Figure 13: Country Factors : United Kingdom

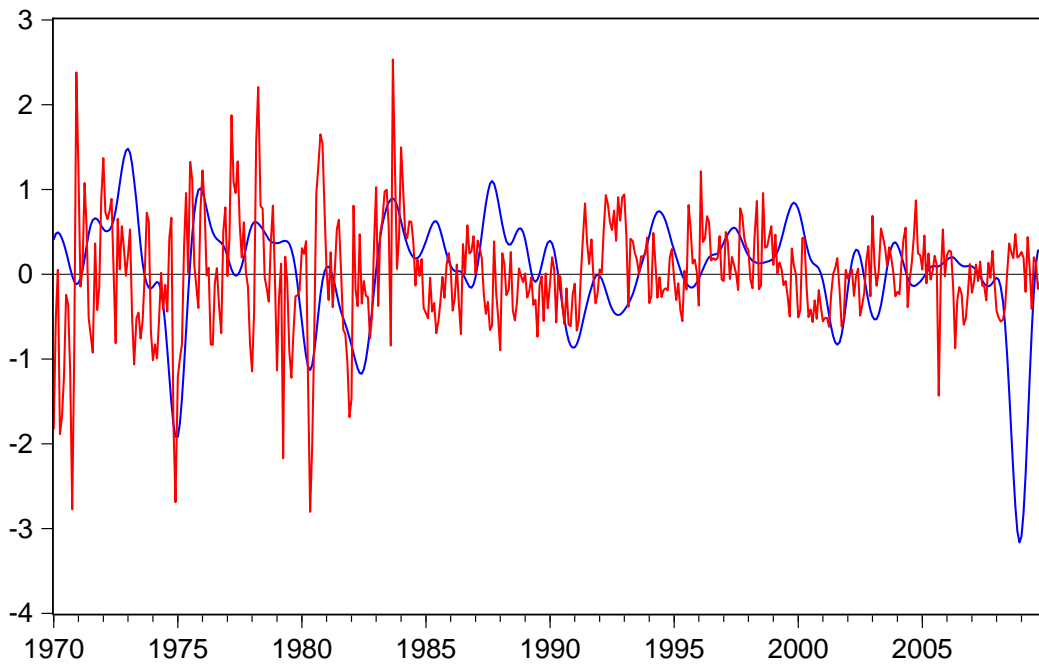
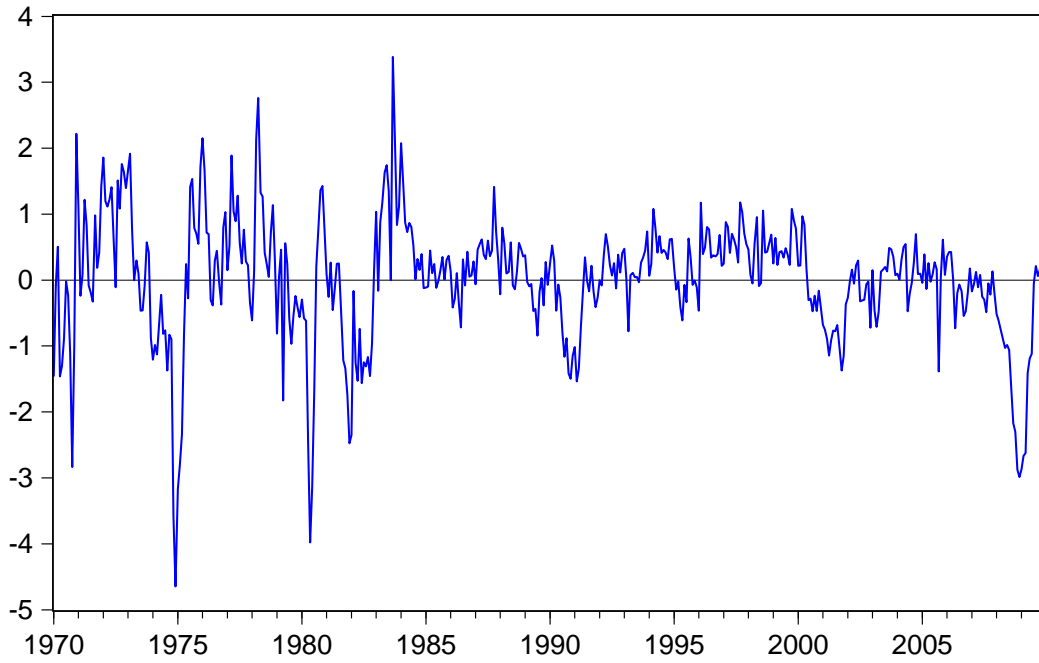
Smoothed Factor



— Common (G-7) Factor
— Idiosyncratic Component

Figure 14: Country Factors : United States

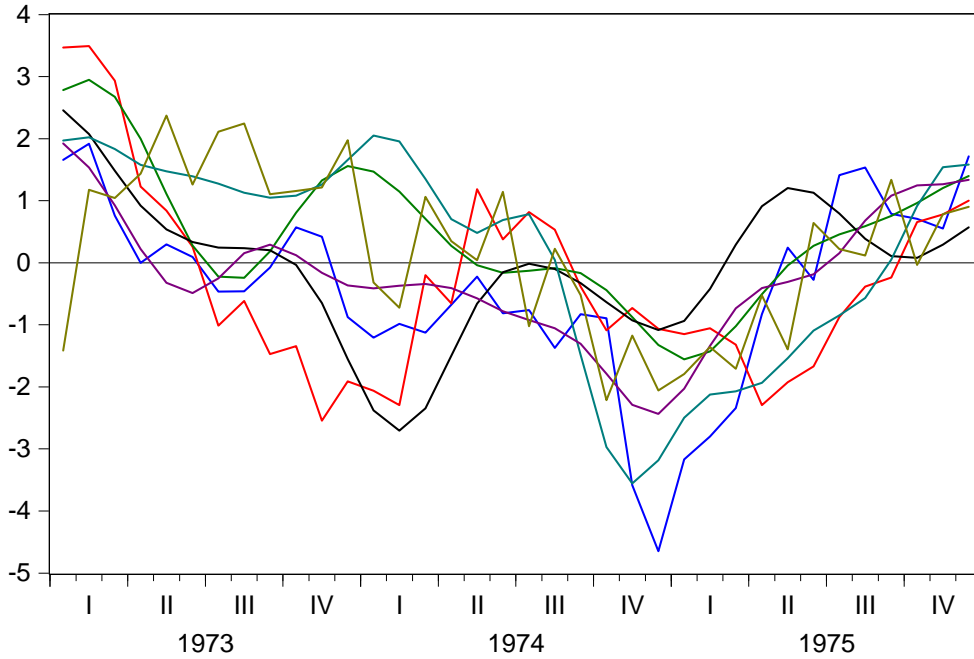
Smoothed Factor



— Common (G-7) Component
— Idiosyncratic Component

Figure 15: Country Factors around 1974 and 2008 Recessions

Country Factors around 1974



Country Factors around 2008

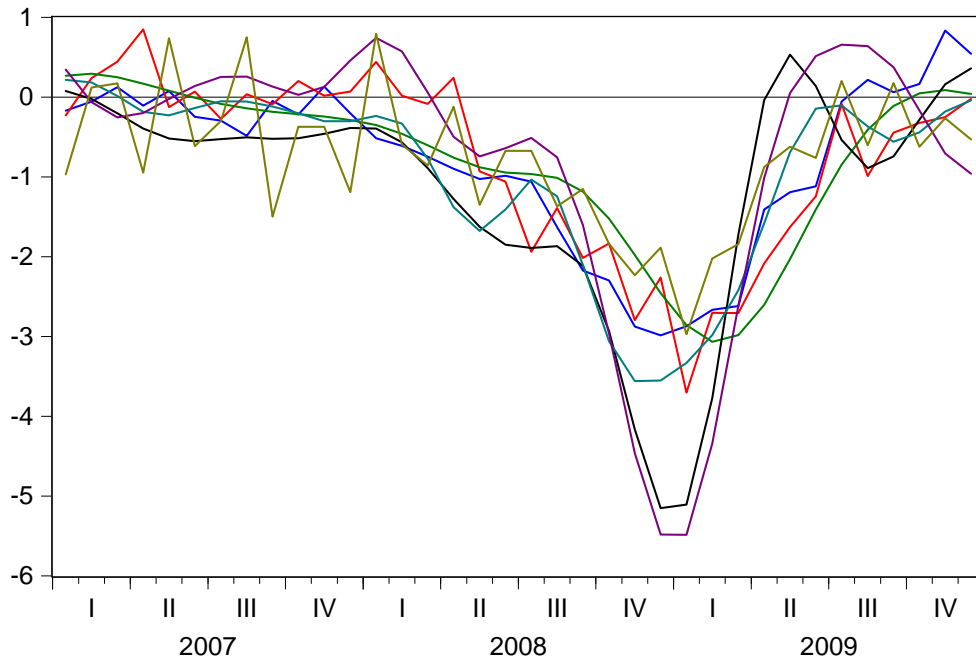
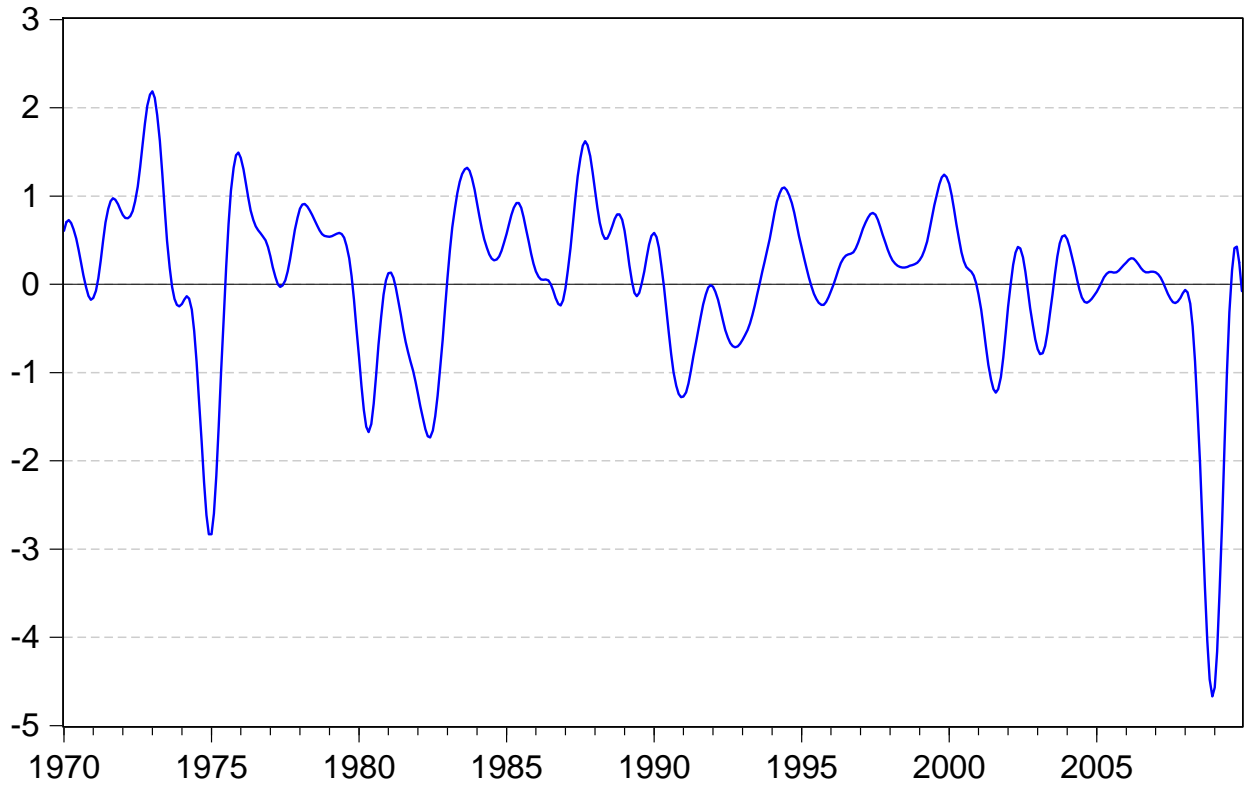


Figure 16: G-7 Factor



Appendix

A Details of the State Space Model for Country Factor Extraction

Consider a generic country with all six indicators observed. Order the indicators as employment (monthly), industrial production (monthly), retail sales (monthly), income (quarterly), GDP (quarterly) and initial claims (monthly). Then the system vectors and matrices in (5) and (6) are:

$$\boldsymbol{\alpha}_t = \left[x_t \quad x_{t-1} \quad x_{t-2} \quad \varepsilon_t^1 \quad \varepsilon_{t-1}^1 \quad \varepsilon_{t-2}^1 \quad \cdots \quad \varepsilon_t^6 \quad \varepsilon_{t-1}^6 \quad \varepsilon_{t-2}^6 \right]' \quad (10)$$

$$\mathbf{y}_t = \left[y_t^1 \quad y_t^2 \quad \cdots \quad y_t^6 \right]' \quad (11)$$

$$\mathbf{c} = \left[c^1 \quad c^2 \quad \cdots \quad c^6 \right]' \quad (12)$$

$$\mathbf{u}_t = \left[\eta_t \quad v_t^1 \quad \cdots \quad v_t^6 \right]' \quad (13)$$

$$\boldsymbol{\epsilon}_t = \left[v_t^1 \quad v_t^2 \quad \cdots \quad v_t^6 \right]' \quad (14)$$

$$\mathbf{T} = \begin{bmatrix} \rho_1 & \rho_2 & \rho_3 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_1^1 & \gamma_2^1 & \gamma_3^1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \gamma_1^6 & \gamma_2^6 & \gamma_3^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix} \quad (15)$$

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}' \quad (16)$$

$$\mathbf{Q} = \text{diag} \left(\sigma_\eta^2 \quad \sigma_{v,1}^2 \quad \cdots \quad \sigma_{v,6}^2 \right) \quad (17)$$

$$\mathbf{Z}' = \begin{bmatrix}
\beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 & \beta_6 \\
0 & 0 & 0 & \beta_4 & \beta_5 & 0 \\
0 & 0 & 0 & \beta_4 & \beta_5 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{18}$$